CSE 431: Theory of Computation (Spring 2008) — **Instructor: James R. Lee**
Problem Set #3 — **Due: Thursday, Apr 24**

Reading assignment: Chapters 4 and 5 of Sipser.

Instructions: Same as homework #1.

This problem set has **four** regular problems worth 10 points each, and one extra credit problem. Please be as careful as possible in your arguments and your answers.

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1. Which of the following problems about Turing machines is decidable and which is not? Briefly justify your answers.

   (a) To determine, given a Turing machine M and a string w, whether M ever moves its head to the left when it is run on input w.

   (b) To determine, given a Turing machine M, whether the tape ever contains four consecutive 1's during the course of M's computation when it is run on input 01.

2. Let A and B be two *disjoint* languages. Say that C **separates** A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

3. (a) Prove that a language A is Turing-recognizable if and only if A is mapping reducible to A_{TM}.

   (b) Prove that a language B is Turing-decidable if and only if B is mapping reducible to $\{0^n 1^n : n \geq 1\}$.

4. Define

$$f(m) = \begin{cases} 3m+1 & \text{for odd } m \\ m/2 & \text{for even } m. \end{cases}$$

   for any natural number m. If you start with a number m and iterate f, you obtain a sequence: $m, f(m), f(f(m)), \ldots$ Stop if you ever hit 1. Extensive computer tests have shown that every starting point m between 1 and 10^{18} eventually gives a sequence that ends in 1. The question of whether this happens for **all** starting points is unsolved, and is called the $3m+1$ conjecture. Paul Erdös offered \$500 for its solution. You don't have to solve the conjecture.

   Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the $3m+1$ conjecture.