CSE 431: Theory of Computation (Spring 2008) **Instructor: James R. Lee**
Problem Set #2 **Due: Thursday, Apr 17**

Reading assignment: Chapters 3 and 4 of Sipser.

Instructions: Same as homework #1.

This problem set has **four** regular problems worth 10 points each, and one extra credit problem. Please be as careful as possible in your arguments and your answers.

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1. Let S be an infinite, Turing-recognizable language. Prove that S has an infinite, decidable subset. (Hint: You might use problem 4 of the first homework.)

2. Define the language

$$A = \Big\{ \langle M \rangle : M \text{ is an NFA that only accepts palindromes in } \{0,1\}^* \Big\}.$$

   (Note that for $\langle M \rangle$ to be in A, it does *not* need to accept all palindromes, it just cannot accept any non-palindromes.)

   Prove that A is **decidable**.

3. (Problem 4.21 in Sipser.) Say that an NFA is *ambiguous* if its accepts some string along two different computation branches. Let $\mathsf{AMBIG}_{\text{NFA}} = \{\langle N \rangle : N \text{ is an ambiguous NFA}\}$. Show that $\mathsf{AMBIG}_{\text{NFA}}$ is decidable.

   [Hint: One elegant way to solve this problem is to construct a suitable DFA and then run E_{DFA} on it (E_{DFA} is defined on page 168 in Chapter 4).]

4. A complex number is *algebraic* if it is the root of a non-zero polynomial with integer coefficients. Show that the set of algebraic numbers countable.

5. **(Extra credit)** Show that single-tape Turing machines that cannot write on the portion of the tape containing the input can only recognize regular languages.