CSE 431 Spring 2007
Assignment #3

Due: Friday, April 20, 2007

Reading assignment: Read Chapter 5 of Sipser's text. We will cover section 5.3 before we cover computation histories in section 5.1.

Problems:

1. Suppose that $A \subseteq \{\langle M\rangle \mid M \text{ is a decider TM}\}$ and that A is Turing-recognizable. Prove that there is a decidable language D such that $D \neq L(M)$ for any M with $\langle M\rangle \in A$. (Hint: You may find it helpful to consider an enumerator for A.)

   (In general it seems hard to tell if a TM is a decider but one might guess that there could be some easy-to-recognize special format for a restricted class of TMs such that (1) any TM in the format must be a decider, and (2) for every decider there is an equivalent TM in this format. The answer to this question rules this out.)

2. Let $L = \{\langle M, w\rangle \mid M$ attempts to move left while on the left end of its tape during its computation on input $w\}$. Prove that L is undecidable.

3. Let $R = \{\langle M, w\rangle \mid M$ attempts to move left at some step of its computation on input $w\}$. Prove that R is decidable.

4. For a string $w \in \{0,1\}^*$, let the *1's-complement* of w, $\overline{w}$, be the string obtained by replacing each 0 of w by a 1 and each 1 of w by a 0.
   Let $C = \{\langle M\rangle \mid M$ is a TM with input alphabet $\{0,1\}$ such that, for every $w \in \{0,1\}^*$, M accepts w if and only if M accepts $\overline{w}\}$. Show that C is undecidable.

5. Show that A is Turing-recognizable if and only if $A \leq_m A_{TM}$.

6. Show that A is decidable if and only if $A \leq_m 0^*1^*$.

7. (Extra credit) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the *busy beaver function* $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of k, consider all k-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not a computable function.