

CSE 431 Spring 2007

Assignment #2

Due: Friday, April 13, 2006

Reading assignment: Read Chapter 4 of Sipser's text.

Problems:

1. Show that both the decidable languages and the Turing recognizable languages are closed under the concatenation, intersection, and star operations.
2. Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case when it is infinite.)
3. Use the result of question 2 to show that any infinite Turing-recognizable language contains an infinite decidable subset.

4. Let $INFINITE_{PDA} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$. Show that $INFINITE_{PDA}$ is decidable.

5. Show that the set of complex numbers,

$$QUADRATIC-ROOT = \{x \in \mathbb{C} \mid \text{there are integers } a \neq 0, b, \text{ and } c \text{ such that } ax^2 + bx + c = 0\}$$

is countable.

6. (Bonus) Let C be a language. Prove that C is Turing-recognizable iff there is a decidable language D such that $C = \{x \mid \exists y \text{ such that } \langle x, y \rangle \in D\}$.