# CSE 431 Spring 2006
# Assignment #3

Due: Friday, April 21, 2006

**Reading assignment:** Read Chapter 5 of Sipser's text. We will cover section 5.3 before we cover computation histories in section 5.1.

**Problems:**

1. Suppose that $A \subseteq \{\langle M \rangle \mid M \text{ is a decider TM}\}$ and that $A$ is Turing-recognizable. (That is, $A$ only contains descriptions of TMs that are deciders but it might not accept all such descriptions.)
   Prove that there is a decidable language $D$ such that $L(M) \neq D$ for any $M$ with $\langle M \rangle \in A$. (Intuitively, this means that one couldn't come up with some restricted easy-to-recognize format for deciders that captured all decidable languages.)
   (Hint: You may find it helpful to consider an enumerator for $A$.)

2. Let $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that $T$ is undecidable.

3. Sipser's text (either edition), Problem 5.13.

4. Show that for all Turing-recognizable problems $A$, $A \leq_m A_{TM}$.

5. Sipser's text (1st edition problem 5.10; 2nd edition problem 5.24.

6. (Bonus) Let $\Gamma = \{0, 1, blank\}$ be the tape alphabet for all TMs in this problem. Define the *busy beaver function* $BB : \mathbb{N} \to \mathbb{N}$ as follows: For each value of $k$, consider all $k$-state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of 1s that remain on the tape among all of these machines. Show that $BB$ is not a computable function.