

# CSE 431 Spring 2006

## Assignment #2

Due: Friday, April 14, 2006

**Reading assignment:** Read Chapter 4 of Sipser's text.

### Problems:

1. Prove that a language is decidable if and only if there is an enumerator that enumerates it in lexicographic order. (Hint: Handle the case where the language is finite separately from the case when it is infinite.)
2. Use the above to show that any infinite Turing-recognizable language contains an infinite decidable subset.
3. Let  $S = \{\langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}$ . Show that  $S$  is decidable.
4. Let  $INFINITE_{PDA} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$ . Show that  $INFINITE_{PDA}$  is decidable.

5. Show that the set of complex numbers,

$$QUADRATIC-ROOT = \{x \in \mathbb{C} \mid \text{there are integers } a \neq 0, b, \text{ and } c \text{ such that } ax^2 + bx + c = 0\}$$

is countable.

6. (Bonus) Let  $C$  be a language. Prove that  $C$  is Turing-recognizable iff there is a decidable language  $D$  such that  $C = \{x \mid \exists y \text{ such that } \langle x, y \rangle \in D\}$ .