CSE 431: Introduction to Theory of Computation Spring 2005

PROBLEM SET 6

Due Friday, May 20, 2005, in class

Reading assignment: Sipser's book, Sections 7.3-7.5.

Instructions: Same as for Problem set 1.

Each question is worth 10 points. Please be as clear and concise as possible in your arguments and answers. The optional problem is for extra credit.

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1. Show that, if P = NP, a polynomial time algorithm exists, that, given a 3SAT instance ϕ, actually produces a satisfying assignment for ϕ if it is satisfiable.

2. Let $U = \{\langle M, x, 1^t \rangle \mid M$ is a nondeterministic Turing machine that accepts x within t steps$\}$. Show that U is NP-complete.

3. Let HALF-INDSET $= \{\langle G \rangle \mid G$ is an undirected graph that has an independent set of size at least $n/2$, where n is the number of vertices in $G\}$. Prove that INDSET $\leq_P$ HALF-INDSET.

4. An HornSAT formula is an AND of clauses, where each clause is an OR of several literals, *at most one of which is a positive literal*. That is, every clause is either of the form $(\overline{x_1} \vee \overline{x_2} \vee \cdots \vee \overline{x_k} \vee y)$ for some $k \geq 0$ or of the form $(\overline{x_1} \vee \overline{x_2} \vee \cdots \vee \overline{x_k})$ for some $k \geq 1$. Define the language

   $$\mathsf{HornSAT} = \{\langle \phi \rangle \mid \phi \text{ is a HornSAT formula that is satisfiable}\} .$$

   Show that $\mathsf{HornSAT} \in$ P.
   (Hint: The clause $(\overline{x_1} \vee \overline{x_2} \vee \cdots \vee \overline{x_k} \vee y)$ is equivalent to the implication $((x_1 \wedge x_2 \wedge \cdots \wedge x_k) \Rightarrow y)$. Now use a conservative approach, setting variables to True only when necessary, and prove that this correctly ascertains whether the formula is satisfiable or not.)

5. * **(Optional Problem)** This problem investigates *resolution*, a method for proving the unsatisfiability of CNF-formulas. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a formula in CNF, where the C_i are its clauses. Let $\mathcal{C} = \{C_i \mid C_i$ is a clause of $\phi\}$. In a *resolution step*, we take two clauses C_a and C_b in $\mathcal{C}$ which both have some variable x, occurring positively in one of the clauses and negatively in the other. Thus $C_a = (x \vee y_1 \vee y_2 \vee \cdots \vee y_k)$ and $C_b = (\bar{x} \vee z_1 \vee z_2 \vee \cdots \vee z_\ell)$, where the y_i and z_i are literals. We form the new clauses $(y_1 \vee y_2 \vee \cdots \vee y_k \vee z_1 \vee z_2 \vee \cdots \vee z_\ell)$ and remove repeated literals. Add this new clause to $\mathcal{C}$. Repeat the resolution steps until no additional clauses can be obtained. If the empty clause () is in $\mathcal{C}$ then declare ϕ unsatisfiable.

   Say that resolution is *sound* if it never declares satisfiable formulas to be unsatisfiable. Say that resolution is *complete* if all unsatisfiable formulas are declared to be unsatisfiable.

   (a) Show that resolution is sound and complete.
   (b) Define 2SAT $= \{\langle \phi \rangle \mid \phi$ is a 2CNF formula that is satisfiable$\}$ (a 2CNF formula is an AND of clauses where each clause is an OR of at most two literals). Use part (a) to show that 2SAT $\in$ P.