CSE 431
Spring Quarter 2001
Assignment 3
Due Friday, April 20

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Use a diagonal argument to show that

$$AH_{TM} = \{\langle M \rangle : M \text{ halts on all inputs}\}$$

is not Turing recognizable. Hint: Your proof by contradiction will assume that there is a Turing enumerator of AH_{TM}. The enumerator outputs the members of AH_{TM} in the order $\langle M_1 \rangle, \langle M_2 \rangle, \ldots$. The possible inputs are members of $\{0,1\}^*$ which can also be indexed $s_1, s_2, \ldots$. Define a Turing machine decider M whose encoding cannot appear in AH_{TM}.

2. (10 points) Let E be an enumerator with the property that if E enumerates an infinite language in the order $w_1, w_2, \ldots$ (the eventual output of E is $w_1\#w_2\#w_3\#\cdots$.) and $|w_i| < |w_{i+1}|$ for all i. Show that the language enumerated is Turing decidable.

3. (10 points) Show that every infinite Turing recognizable language has an infinite Turing decidable subset. Hint: Use 2 above.