

LIS So Far

0	1	2	3	4	5	6	7
5	-6	3	6	-5	2	8	10

$LIS(i, j)$ is "Number of elements of the maximum increasing subsequence from $0, \dots, i$ where every element of the sequence is at most $A[j]$ "

Need a recurrence

$$LIS(i, j) = \begin{cases} 0 & \text{if } i < 0 \\ \mathbb{I}[A[i] \leq A[j]] & \text{if } i = 0 \\ LIS(i-1, j) & \text{if } A[i] > A[j] \\ \max\{1 + LIS(i-1, i), LIS(i-1, j)\} & \text{otherwise} \end{cases}$$

"Indicator" kind of like 312: cast bool to int.

If $A[i] > A[j]$ element i cannot be included in an increasing subsequence where every element is at most $A[j]$. So taking the largest among the first $i-1$ suffices.

If $A[i] \leq A[j]$, then if we include i , we may include elements to the left only if they are less than $A[i]$. (since $A[i]$ will now be the last, and therefore largest, of elements $0 \dots i$. If we don't include i we want the maximum increasing subsequence among $0 \dots i-1$.)

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LIS (activity)

$$LIS(i, j) = \begin{cases} 0 & \text{if } i < 0 \\ \mathbb{I}[A[i] \leq A[j]] & \text{if } i = 0 \\ LIS(i-1, j) & \text{if } A[i] > A[j] \\ \max\{1 + LIS(i-1, i), LIS(i-1, j)\} & \text{otherwise} \end{cases}$$

	0, 5	1, -6	2, 3	3, 6	4, -5	5, 2	6, 8	7, 10
0, 5	1	0	0	1	0	0	1	1
1, -6	1	1	1	1	1	1	1	1
2, 3	2							
3, 6								
4, -5								
5, 2								
6, 8								
7, 10								

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Example

What's the distance between babyodas and tastysoda?

x	B	A	B		Y	Y	O	D	A	S
op	Sub		sub	ins		sub				del
y	T	A	S	T	Y	S	O	D	A	

Distance: 5, one point for each colored box

Quick Checks – can you explain these?

If x has length n and y has length m , the edit distance is at most $\max(x, y)$

The distance from x to y is the same as from y to x (i.e. transforming x to y and y to x are the same)

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Computing (9)

Fill in the next two entries. Be careful with the sub/match distinction!

OPT(i, j)	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	4	3	3	4	5	6	7	8
Y 5	5	5	5	4	4	3				
S 6										
O 7										
D 8										
A 9										

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