

Greedy MST algorithms

You've seen two algorithms for MSTs

Kruskal's Algorithm:

Order: Sort the edges in increasing weight order

Rule: If connect new vertices (doesn't form a cycle), add the edge.

Prim's Algorithm:

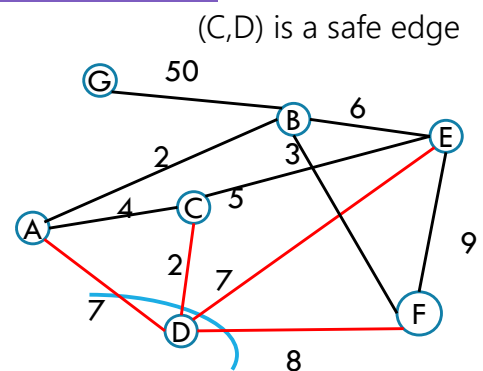
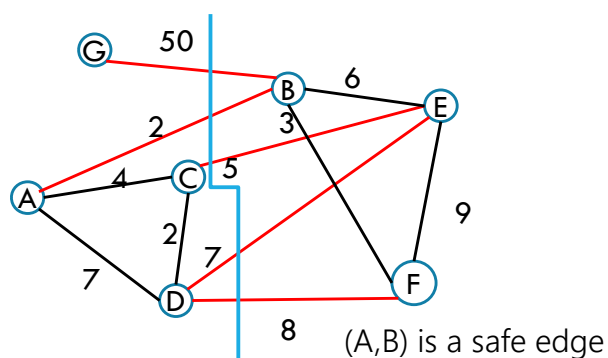
Order: lightest weight edge that adds a new vertex to our current component

Rule: Just add it!

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Safe Edge

Call an edge, e , a "safe edge" if there is some cut $(S, V \setminus S)$ where e is the minimum edge spanning that cut



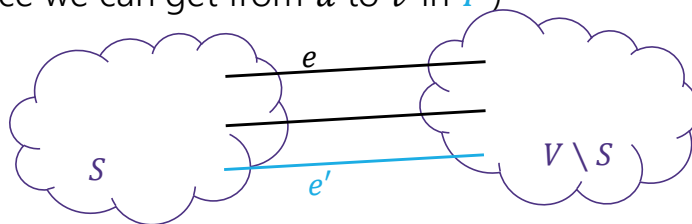
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MSTs and Safe Edges

Claim: Every safe edge is in the MST.

Proof: Suppose, for the sake of contradiction, that $e = (u, v)$ is a safe edge, but not in the MST.

Let $(S, V \setminus S)$ be a cut where e is the minimum edge spanning $(S, V \setminus S)$. Let T' be the MST. The MST has (at least one) an edge e' that crosses the cut (since we can get from u to v in T')



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Trip Planning (definition)

Your goal is to follow a pre-set route from New York to Los Angeles.

You can drive 500 miles in a day, but you need to make sure you can stop at a hotel every night (all possibilities premarked on your map)

You'd like to stop for the fewest number of nights possible – what should you plan?

Greedy: Go as far as you can every night.

Is greedy optimal?

Or is there some reason to “stop short” that might let you go further the next night?

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