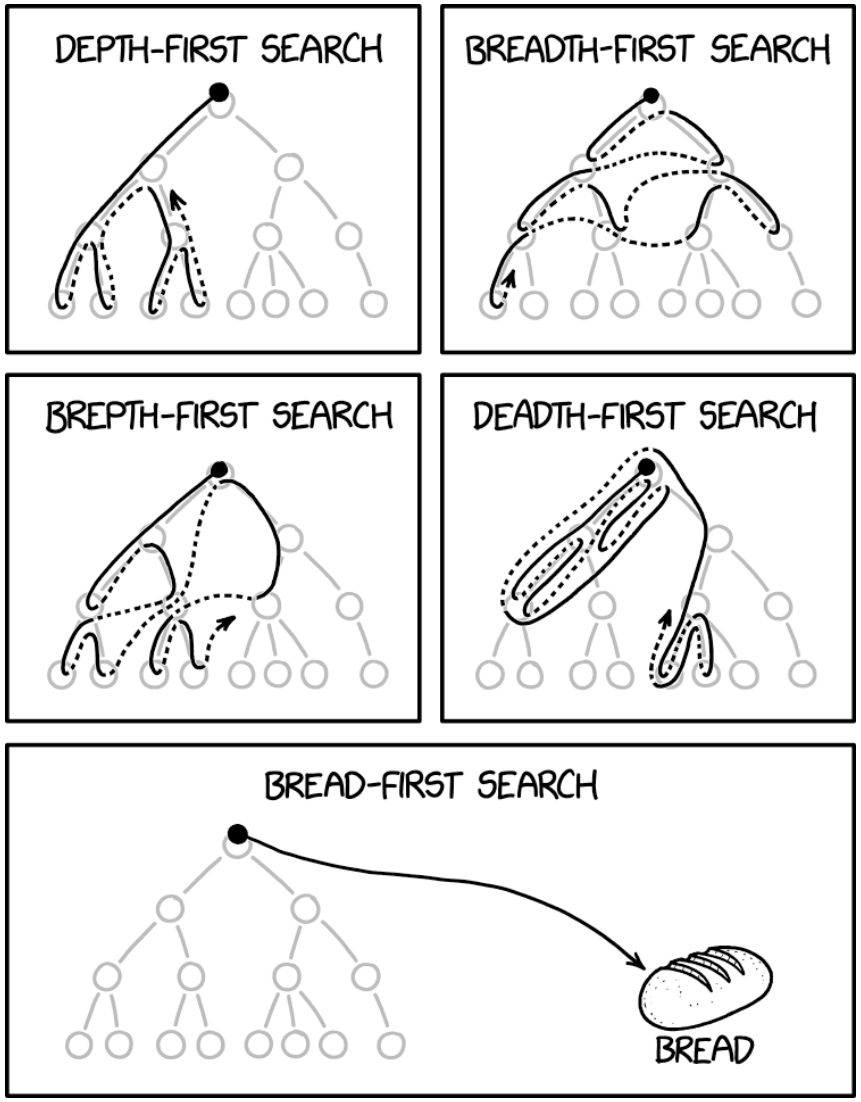


xkcd.com/2407

BFS, DFS

CSE 421 Spring 2026
Lecture 4

Announcement

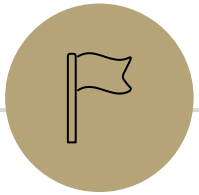
HW1 due Wednesday

Expect it to take longer than usual to submit to gradescope.

To count late problems, we have separate boxes for each problem, so you need to upload the pdf 3 times.

We **strongly** recommend making 1 pdf with all the problems and then selecting pages (way easier for us to fix a mis-done upload).

Just please select pages.



Graphs

332 review

Graphs (basics)

Represent data points and the relationships between them.

That's vague.

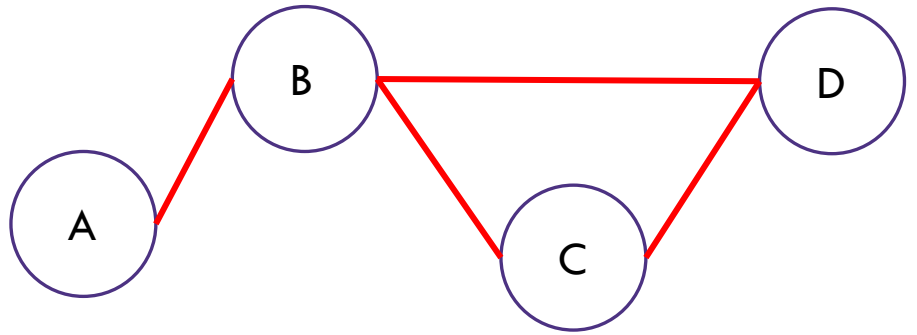
Formally:

A graph is a pair: G = (V,E)

V: set of **vertices** (aka **nodes**)

E: set of **edges**

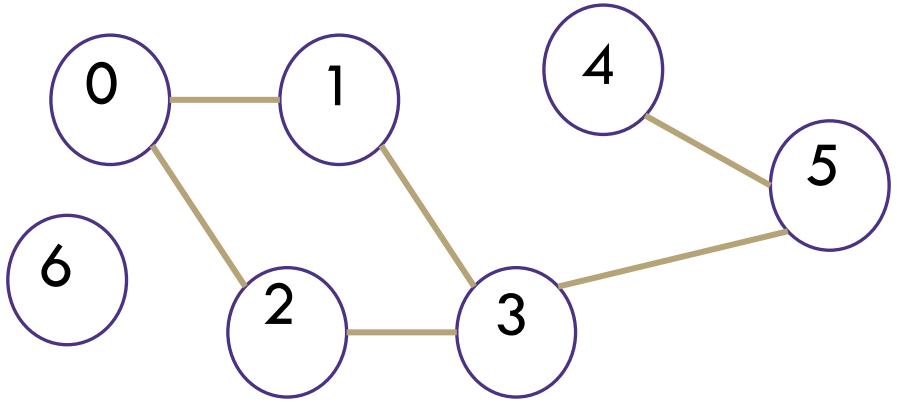
Each edge is a pair of vertices.



$\{A, B, C, D\}$

$\{(A, B), (B, C), (B, D), (C, D)\}$

Adjacency Matrix



In an adjacency matrix a[u][v] is 1 if there is an edge (u,v), and 0 otherwise.

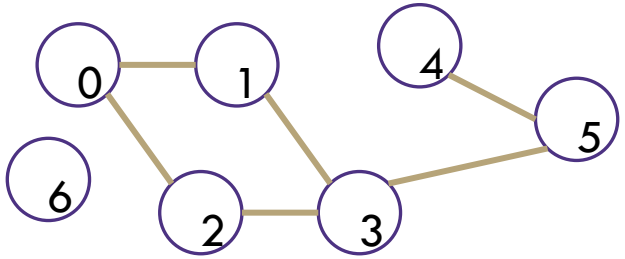
Worst-case Time Complexity ($|V| = n$, $|E| = m$):

- Add Edge: $\Theta(1)$
- Remove Edge: $\Theta(1)$
- Check edge exists from (u,v): $\Theta(1)$
- Get outneighbors of u: $\Theta(n)$
- Get inneighbors of u: $\Theta(n)$

Space Complexity: $\Theta(n^2)$

	0	1	2	3	4	5	6
0	0	1	1	0	0	0	0
1	1	0	0	1	0	0	0
2	1	0	0	1	0	0	0
3	0	1	1	0	0	1	0
4	0	0	0	0	0	1	0
5	0	0	0	1	1	0	0
6	0	0	0	0	0	0	0

Adjacency List

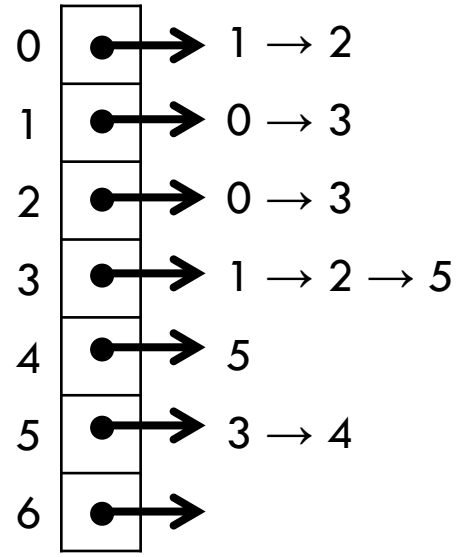


An array where the u^{th} element contains a list of neighbors of u.

Directed graphs: list of out-neighbors (a[u] has v for all (u,v) in E)

Time Complexity ($|V| = n$, $|E| = m$):

- Add Edge: $\Theta(1)$
- Remove Edge: $\Theta(1)$
- Check edge exists from (u,v): $\Theta(1)$
- Get neighbors of u (out): $\Theta(\deg(u))$
- Get neighbors of u (in): $\Theta(n)$



Assume we have hash tables AND linked lists

Space Complexity: $\Theta(n + m)$

Tradeoffs

Adjacency Matrices take more space, and have slower $\Theta()$ bounds, why would you use them?

For **dense** graphs (where m is close to n^2), the running times will be close

And the constant factors can be much better for matrices than for lists.

Sometimes the matrix itself is useful ("spectral graph theory")

For this class, unless we say otherwise, we'll assume we're using Adjacency Lists and the following operations are all $\Theta(1)$

Checking if an edge exists.

Getting the next edge leaving u (when iterating over them all)

"following" an edge (getting access to the other vertex)

To make this work, we usually assume the vertices are numbered.

Graph Algorithms

From 332 you already know:

How to find a topological sort

Use Dijkstra's Algorithm to find shortest paths in (positively) weighted graphs

Use Prim's and Kruskal's Algorithms to find minimum spanning trees.

Depending on which quarter you took 332, you also know:

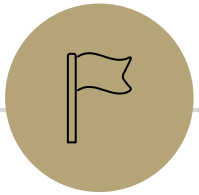
BFS, and at least one application.

That might have been 2-coloring, unweighted shortest paths, or something else.

DFS, and at least one application.

Likely cycle detection, but it might have been something else.

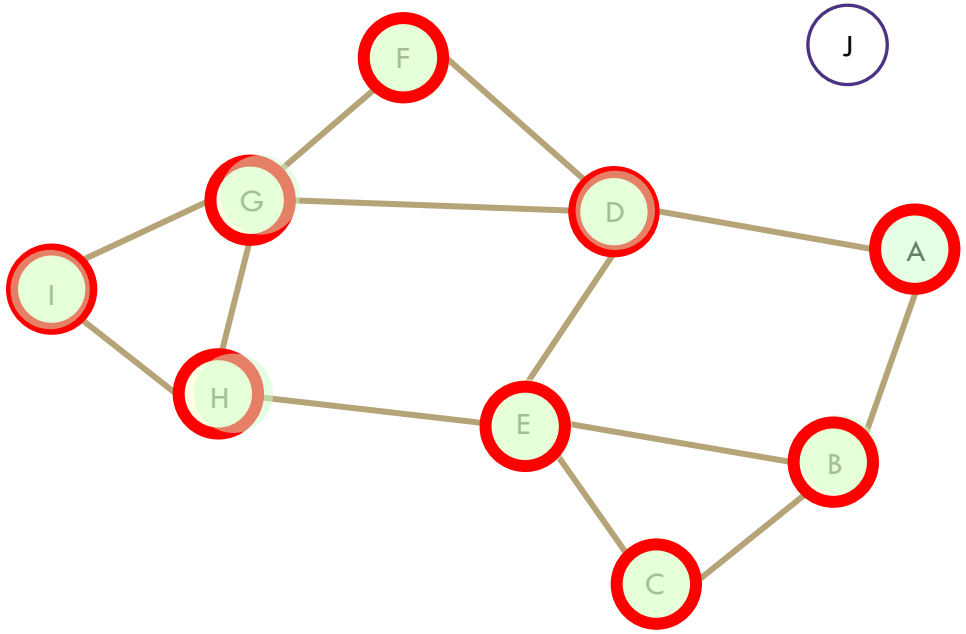
Our goal is ***not*** to memorize algorithms! Our goal is to solve new problems. Even if you've seen these applications, we're coming from a new angle this week.



Traversals

Breadth First Search

```
search(graph)
    toVisit.enqueue(first vertex)
      mark first vertex as seen
    while(toVisit is not empty)
       current = toVisit.dequeue()
       for (v : current.neighbors())
          if (v is not seen)
             mark v as seen
             toVisit.enqueue(v)
```



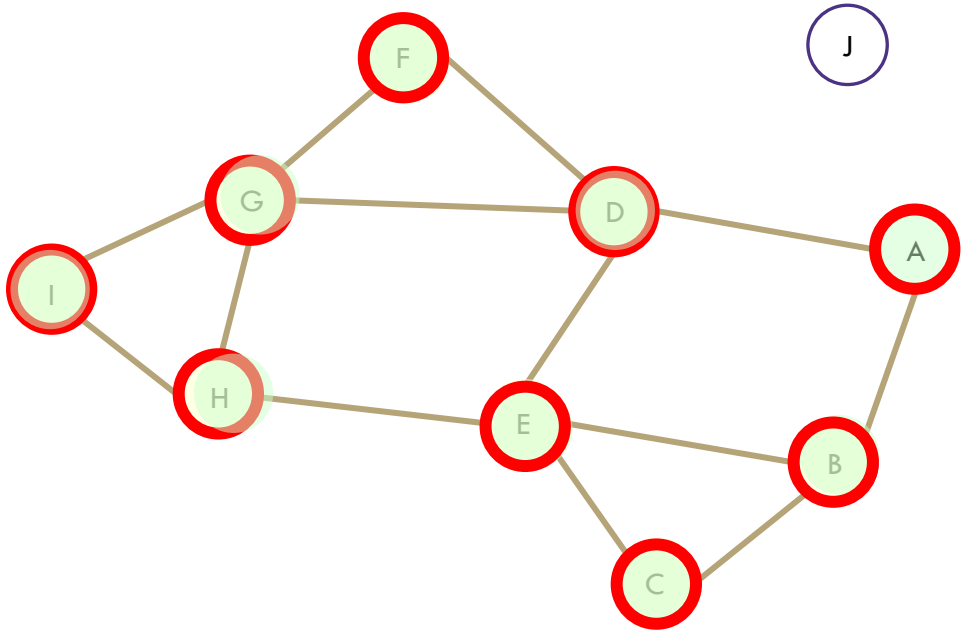
Current node: I

Queue: B D E C F G H I

Finished: A B D E C F G H I

Breadth First Search (example)

```
search(graph)
   toVisit.enqueue(first vertex)
     mark first vertex as seen
   while(toVisit is not empty)
      current = toVisit.dequeue()
      for (v : current.neighbors())
         if (v is not seen)
            mark v as seen
            toVisit.enqueue(v)
```



Hey we missed something…

We're only going to find vertices we can "reach" from our starting point.

If you need to visit everything, just start BFS again somewhere you haven't visited until you've found everything.

Running Time

```
search(graph)
    toVisit.enqueue(first vertex)
      mark first vertex as seen
    while(toVisit is not empty)
       current = toVisit.dequeue()
       for (v : current.neighbors())
          if (v is not seen)
             mark v as seen
             toVisit.enqueue(v)
```

This code might look like:
a loop that goes around m times
Inside a loop that goes around n times,
So you might say $O(mn)$.

That bound is not tight,
Don't think about the loops, think about what happens overall.
How many times is `current` changed?
How many times does an edge get used to define `current.neighbors`?

We visit each vertex at most twice, and each edge at most once: $\Theta(|V| + |E|)$

Old Breadth-First Search Application

Shortest paths in an **unweighted** graph.

Finding the connected components of an **undirected** graph.

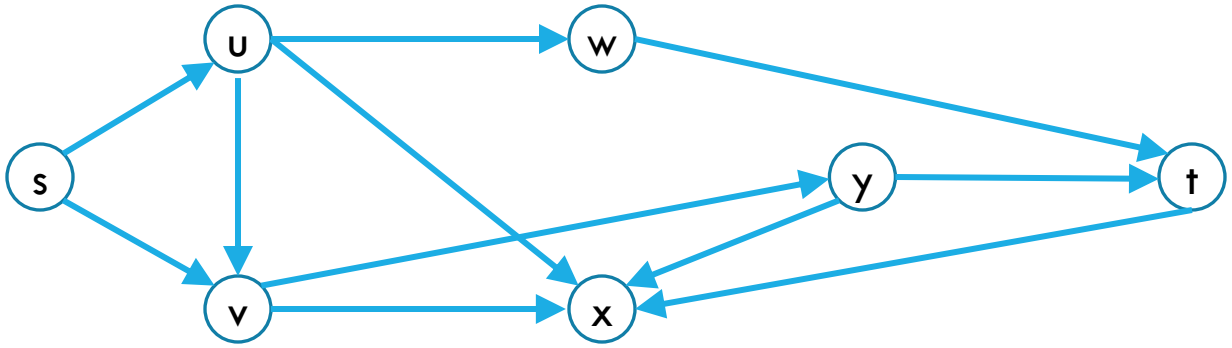
Both run in $\Theta(m + n)$ time,

where m is the number of edges (also written E or $|E|$)

And n is the number of vertices (also written V or $|V|$)

Unweighted Graphs

If the graph is unweighted, how do we find a shortest paths?



What's the shortest path from s to s?
Well….we're already there.

What's the shortest path from s to u or v?
Just go on the edge from s

From s to w,x, or y?
Can't get there directly from s, if we want a length 2 path, have to go through u or v.

A detailed application

Bipartite (also called "2-colorable")

A graph is bipartite (also called 2-colorable) if the vertex set can be divided into two sets V_1, V_2 such that the only edges go between V_1 and V_2.

Called "2-colorable" because you can "color" V_1 red and V_2 blue, and no edge connects vertices of the same color.

We'll adapt BFS to find if a graph is bipartite

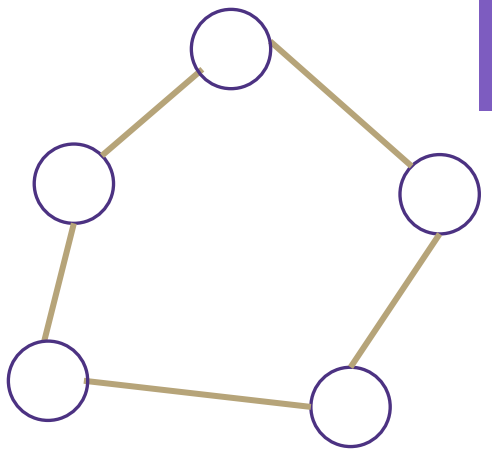
And prove a graph theory result along the way.

A detailed application (example)

Bipartite (also called "2-colorable")

A graph is bipartite (also called 2-colorable) if the vertex set can be divided into two sets V_1, V_2 such that the only edges go between V_1 and V_2.

Called "2-colorable" because you can "color" V_1 red and V_2 blue, and no edge connects vertices of the same color.



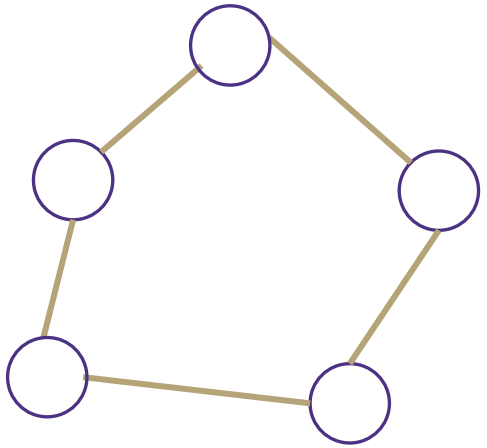
If a graph contains an odd cycle, then it is not bipartite.

Try the example on the right, then proving the general theorem in the light purple box.

Help Robbie figure out how long to make the explanation
Pollev.com/robbie

Lemma 1

If a graph contains an odd cycle, then it is not bipartite.



Start from any vertex, and give it either color.
Its neighbors **must** be the other color.
Their neighbors must be the first color
…
The last two vertices (which are adjacent) must be the same color.
Uh-oh.

BFS with Layers

Why did BFS find distances in unweighted graphs?

You started from u (“layer 0”)

Then you visited the neighbors of u (“layer 1”)

Then the neighbors of the neighbors of u, that weren’t already visited (“layer 2”)

…

The neighbors of layer $i-1$, that weren’t already visited (“layer i”)

BFS With Layers (pseudocode

```
search(graph)
    toVisit.enqueue(first vertex)
    mark first vertex as seen
    toVisit.enqueue(end-of-layer-marker)
    l=1
    while(toVisit is not empty)
        current = toVisit.dequeue()
        if(current == end-of-layer-marker)
            l++
            toVisit.enqueue(end-of-layer-marker)
        current.layer = l
        for (v : current.neighbors())
            if (v is not seen)
                mark v as seen
                toVisit.enqueue(v)
```

It's just BFS!

With some extra bells and whistles.

Layers

Can we have an edge that goes from layer i to layer $i + 2$ (or lower)?

No! If u is in layer i, then we processed its edge while building layer $i + 1$, so the neighbor is no lower than layer i.

Can you have an edge within a layer?

Yes! If u and v are neighbors and both have a neighbor in layer i, they both end up in layer $i + 1$ (from their other neighbor) before the edge between them can be processed.

Testing Bipartiteness

How can we use BFS with layers to check if a graph is 2-colorable?

Well, neighbors have to be “the other color”

Where are your neighbors?

Hopefully in the next layer or previous layer…

Color all the odd layers red and even layers blue.

Does this work?

Lemma 2

If BFS has an intra-layer edge, then the graph has an odd-length cycle.

An “intra-layer” edge is an edge “within” a layer.

Lemma 2 (pf)

If BFS has an intra-layer edge, then the graph has an odd-length cycle.

An "intra-layer" edge is an edge "within" a layer.

Follow the "predecessors" back up, layer by layer.

Eventually we end up with the two vertices having the same predecessor in some level (when you hit layer 1, there's only one vertex)

Since we had two vertices per layer until we found the common vertex, we have $2k + 1$ vertices – that's an odd number!

Lemma 3

If a graph has no odd-length cycles, then it is bipartite.

Prove it by **contrapositive**

We want to show "if a graph is not bipartite, then it has an odd-length cycle.

Suppose G is not bipartite. Then the coloring attempt by BFS-coloring must fail.

Edges between layers can't cause failure – there must be an intra-level edge causing failure. By Lemma 2, we have an odd cycle.

The Big Result

Bipartite (also called "2-colorable")

A graph is bipartite if and only if it has no odd cycles.

Proof:
Lemma 1 says if a graph has an odd cycle, then it's not bipartite (or in contrapositive form, if a graph is bipartite, then it has no odd cycles)

Lemma 3 says if a graph has no odd cycles then it is bipartite.

Lemma 1: If a graph contains an odd cycle, then it is not bipartite.

Lemma 3: If a graph has no odd-length cycles, then it is bipartite.

Algorithm -> proof

The final theorem statement doesn't know about the algorithm – we used the algorithm to prove a graph theory fact!

Wrapping it up

```
BipartiteCheck(graph) //assumes graph is connected!
    toVisit.enqueue(first vertex)
    mark first vertex as seen
    toVisit.enqueue(end-of-layer-marker)
    l=1
    while(toVisit is not empty)
        current = toVisit.dequeue()
          if(current == end-of-layer-marker)
                l++
             toVisit.enqueue(end-of-layer-marker)
        current.layer = l
        for (v : current.neighbors())
            if (v is not seen)
                  mark v as seen
               toVisit.enqueue(v)
            else //v is seen
               if(v.layer == current.layer)
                     return "not bipartite" //intra-level edge
  return "bipartite" //no intra-level edges
```

On homework, you can tell us "assume BipartiteCheck was modified to handle disconnected graphs" if you want those handled automatically.
You just add a wrapper, like you've seen in 332.

Testing Bipartiteness (two statements)

Our algorithm should answer "yes" or "no"

"yes G is bipartite" or "no G isn't bipartite"

Whenever this happens, you'll have two parts to the proof:

If the right answer is yes, then the algorithm says yes.

If the right answer is no, then the algorithm says no.

OR

If the right answer is yes, then the algorithm says yes.
If the algorithm says yes, then the right answer is yes.

Proving Algorithm Correct

If the graph is bipartite, then by Lemma 1 there is no odd cycle. So by the contrapositive of lemma 2, we get no intra-level edges when we run BFS, thus the algorithm (correctly) returns the graph is bipartite.

If the algorithm returns that the graph is bipartite, then we cannot have any intra-level edges (since we check every edge in the course of the algorithm). We proved earlier that there are no edges skipping more than one level. So if we assign odd levels to “red” and even levels to “blue” the algorithm has verified that there are no edges between vertices of the same color. So the graph is bipartite by definition.

DFS vs. BFS

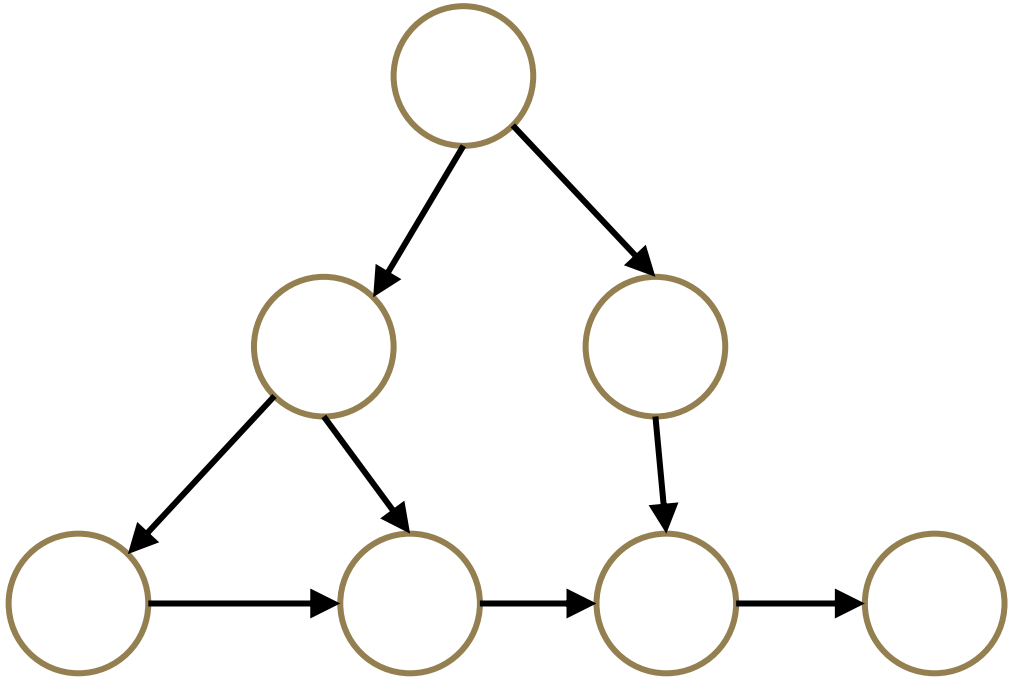
In BFS, we explored a graph "level-wise"

We explored everything next to the starting vertex.

Then we explored everything one step further away.

Then everything one step further

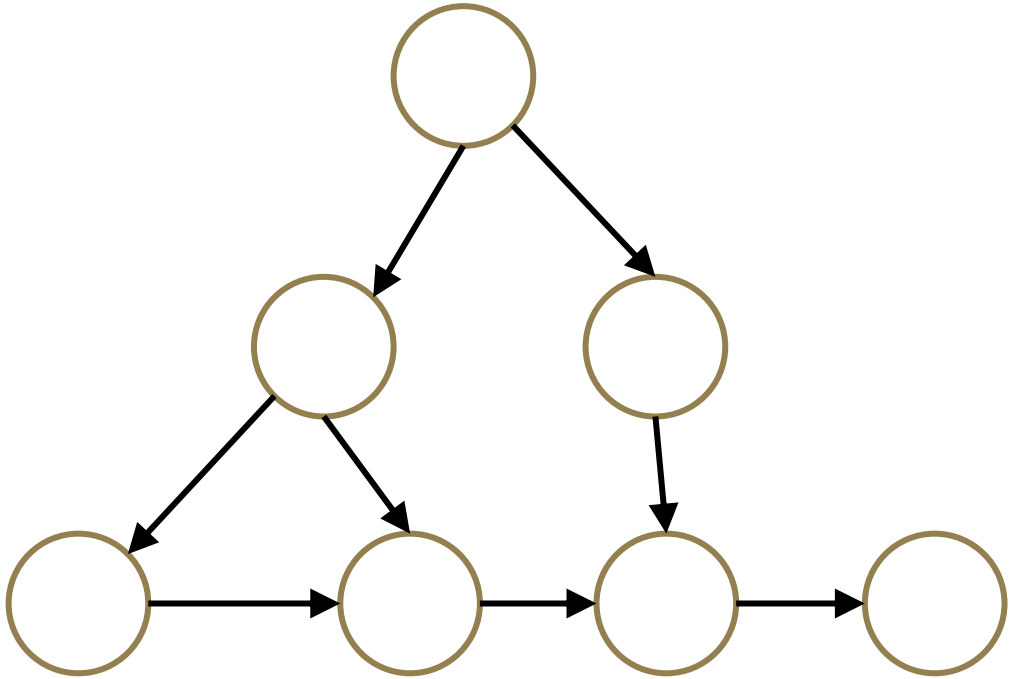
…



DFS vs. BFS (DFS example)

In DFS, we explore deep into the graph.

We try to find new (undiscovered) nodes, then "backtrack" when we're out of new ones.



DFS – pseudocode

In 332, you might have seen two versions: an iterative version (like BFS, but replace the queue with a stack) and a recursive version (use the call stack to manage the search).

The iterative version is a really nice object lesson in data structures.

No one actually writes DFS that way (except in data structures courses).

You'll basically always see the recursive version instead. (using the call stack instead of a user-created stack data structure)

DFS – pseudocode

Instead of using an explicit stack, we're going to use recursion

The call stack is going to be our stack.

We want to explore as deeply as possible from each of our outgoing edges

```
DFS(u)
      Mark u as "seen"
      For each edge (u,v) //leaving u
            If v is not "seen"
                  DFS(v)
            End If
      End For
```

DFS – iterative vs. recursive

Both the explicit stack version and the recursive version “are” DFS.

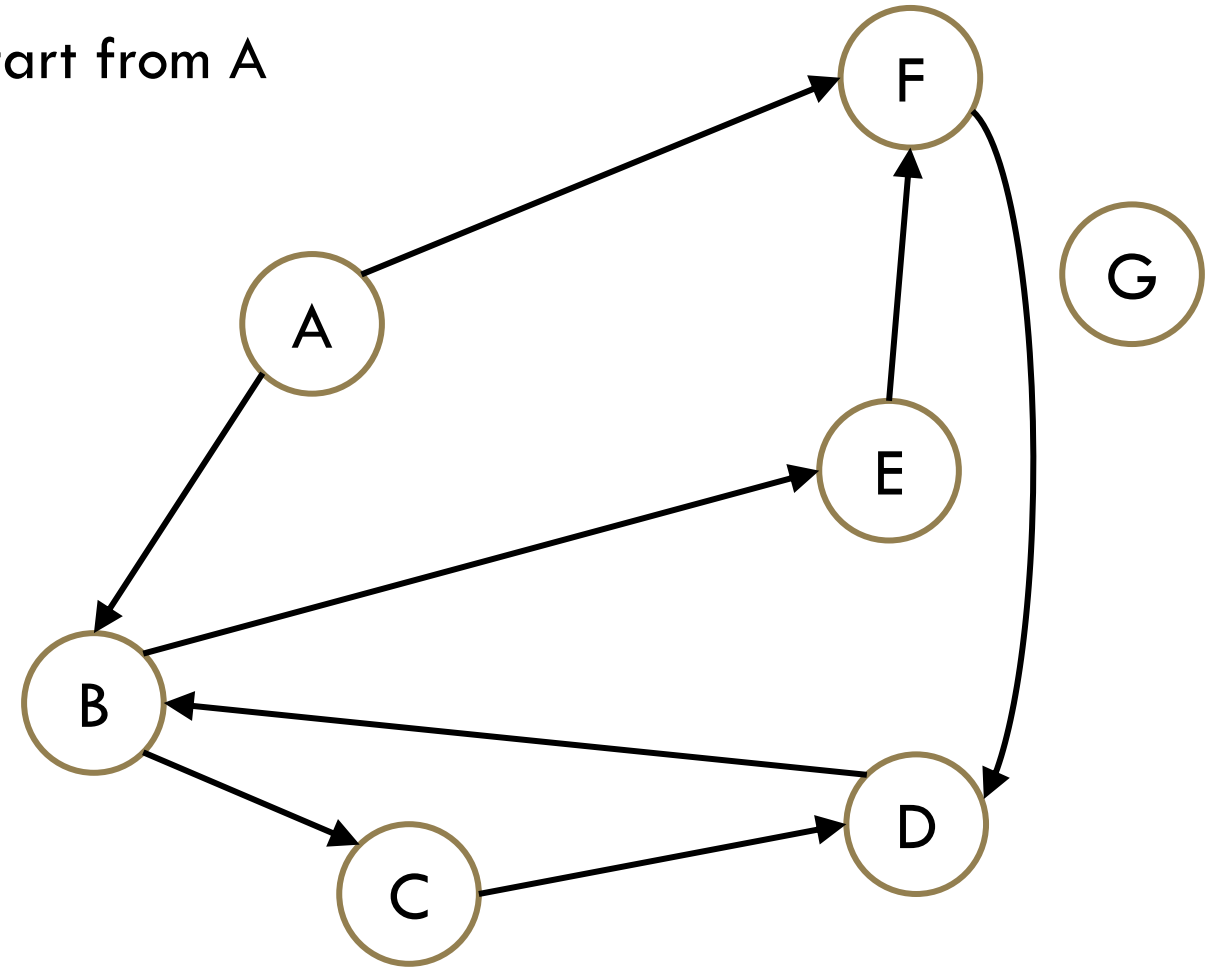
For example, they can both traverse through the graph in the same fundamental way. You can use them for similar applications.

But they’re not identical – they actually use the stack in different ways. If you’re trying to convert from one to the other, you’ll have to think carefully to do it.

Running DFS

```
DFS(u)
      Mark u as "seen"
      For each edge (u,v) //leaving u
            If v is not "seen"
                  DFS(v)
            End If
      End For
```

Start from A



Vertex: F
Last edge used: (F,D)

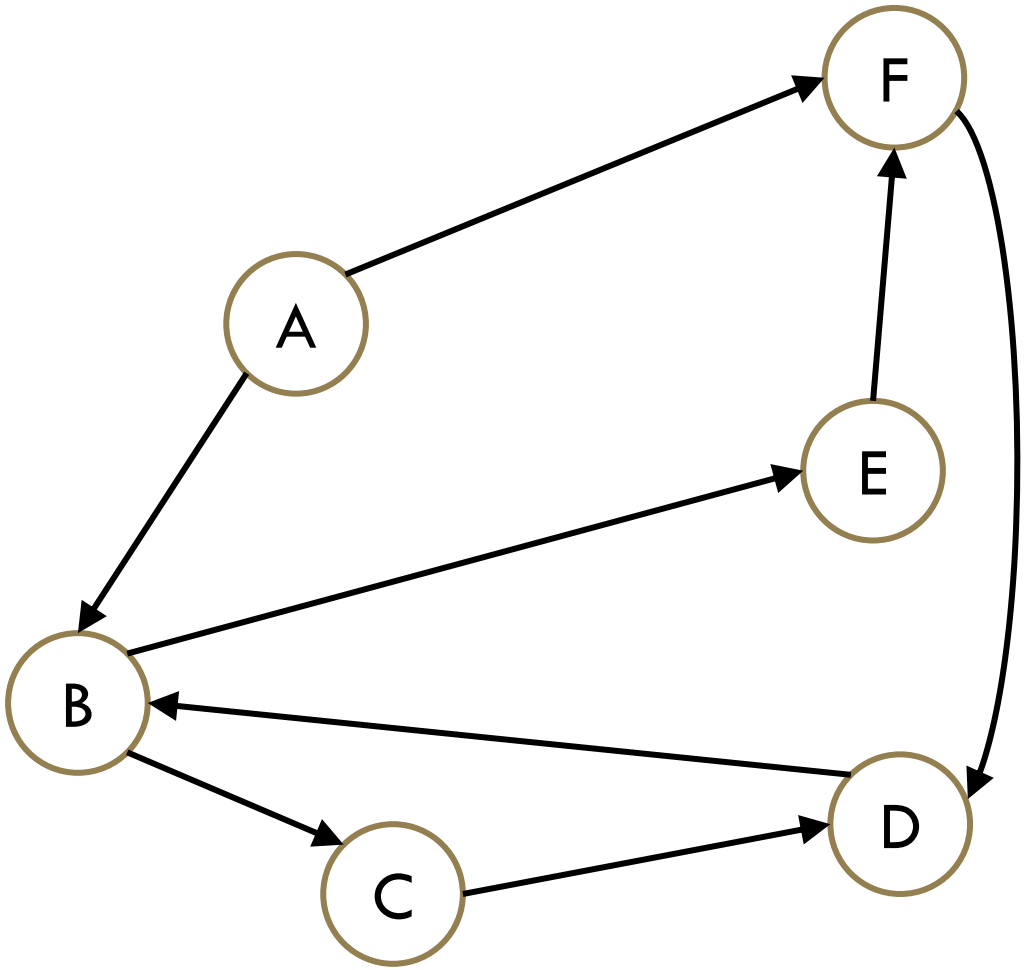
Vertex: E
Last edge used: (E,F)

Vertex: B
Last edge used: (B,E)

Vertex: A
Last edge used: (A,F)

DFS Discovery

```
DFS(u)
    Mark u as "seen"
    For each edge (u,v) //leaving u
        If v is not "seen"
            DFS(v)
        End If
    End For
```



G HEY!

We missed something!

DFS discovery

`DFS(v)` finds exactly the (unseen) vertices reachable from v.

Reaching Everything

One possible use of DFS is visiting every vertex

How can we make sure that happens?

What did you do for BFS when you had this problem?

Add a while loop, and call DFS from each vertex.

```
DFSWrapper(G)
      For each vertex u of G
            If u is not “seen”
                  DFS(u)
            End If
      End For
```

```
DFS(u)
      Mark u as “seen”
      For each edge (u,v) //leaving u
            If v is not “seen”
                  DFS(v)
            End If
      End For
```

Bells and Whistles

Depending on your application, you may add a few extra lines to the DFS code to compute the thing you want.

Usually just an extra variable or two per vertex.

For today's application, we need to know what order vertices come onto and off of the stack.

```
DFS(u)
    Mark u as "seen"
    u.start = counter++
    For each edge (u,v) //leaving u
        If v is not "seen"
            DFS(v)
        End If
    End For
    u.end = counter++
```

```
DFSWrapper(G)
    counter = 1
    For each vertex u of G
        If u is not "seen"
            DFS(u)
        End If
    End For
```

Edge Classification

When we use DFS to search through a graph, we'll have different "kinds" of edges.

Like when we did BFS, we had:

- Edges that went from level i to level $i + 1$
- Intra-level edges.

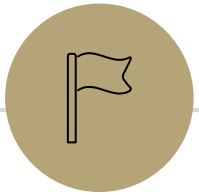
DFS is a bit more complicated.

You're not responsible for the details.

What we want you to know:

"pre" and "post" (aka "stop"/"end") times when you come onto and off of the stack

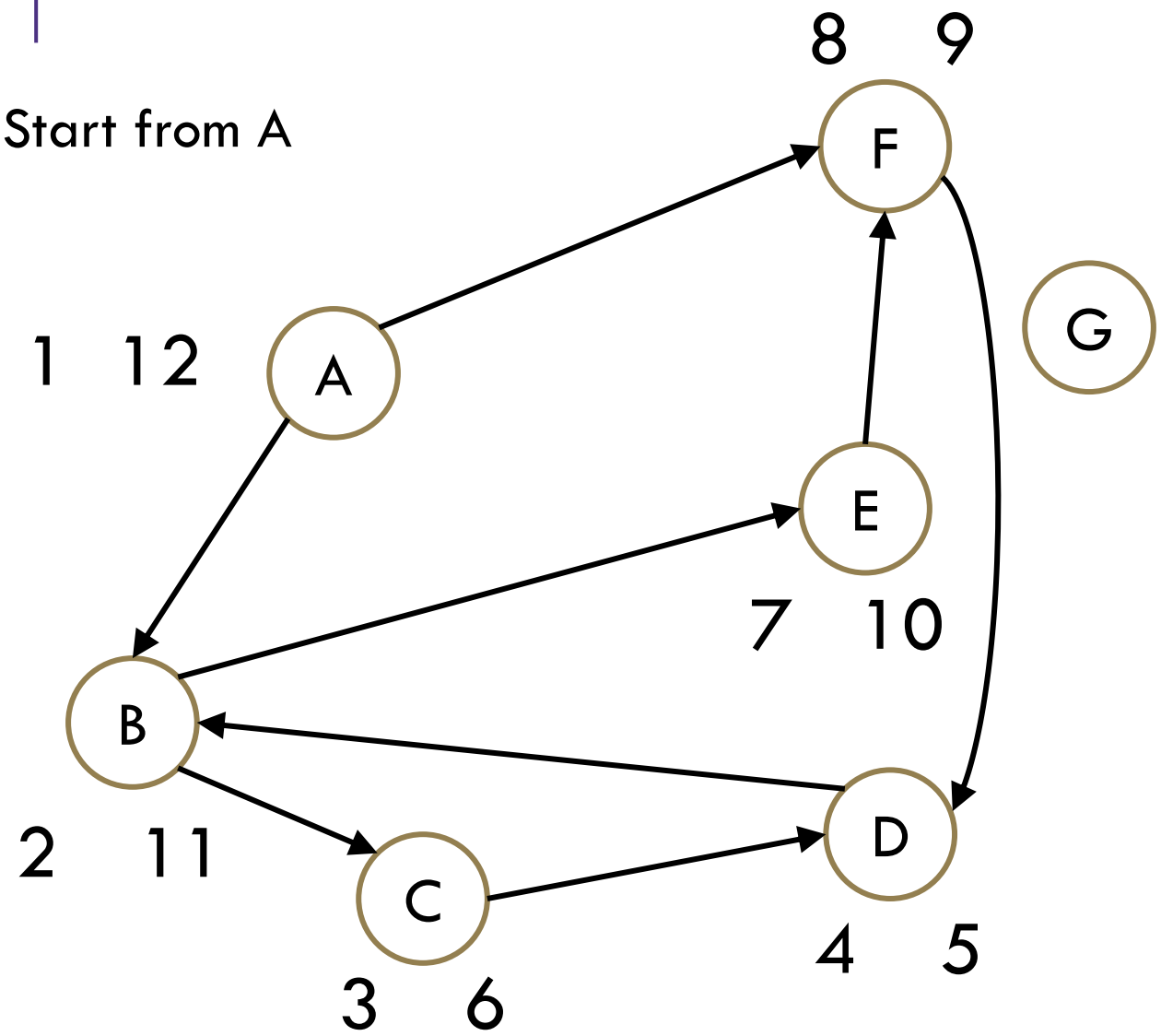
Will tell you what "type" of edge you are



A lot of Details: DFS

DFS: start/end

Start from A



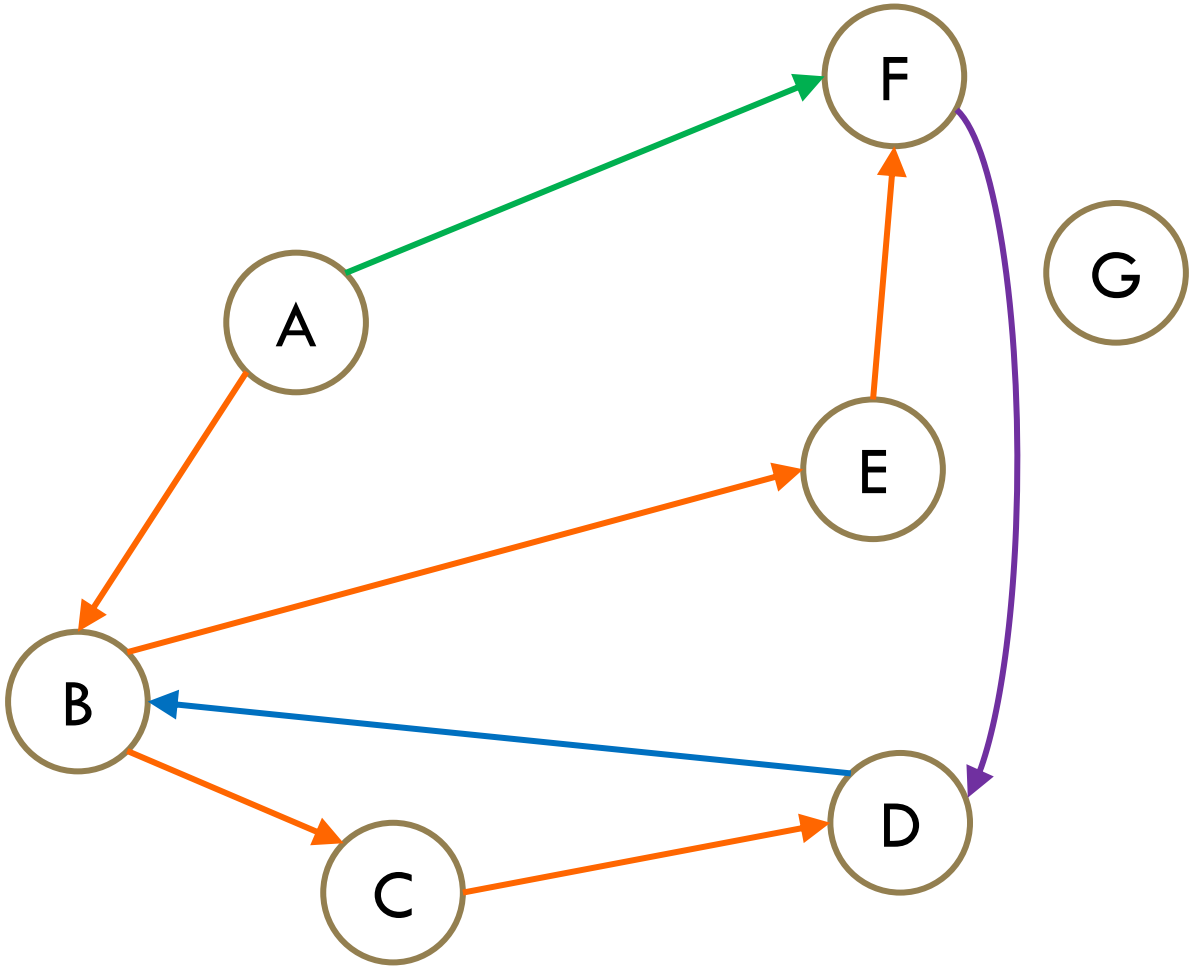

```
DFS(u)
        Mark u as "seen"
        u.start = counter++
        For each edge (u,v) //leaving u
              If v is not "seen"
                    DFS(v)
              End If
        End For
        u.end = counter++
```

Vertex: F
Last edge used: (F,D)

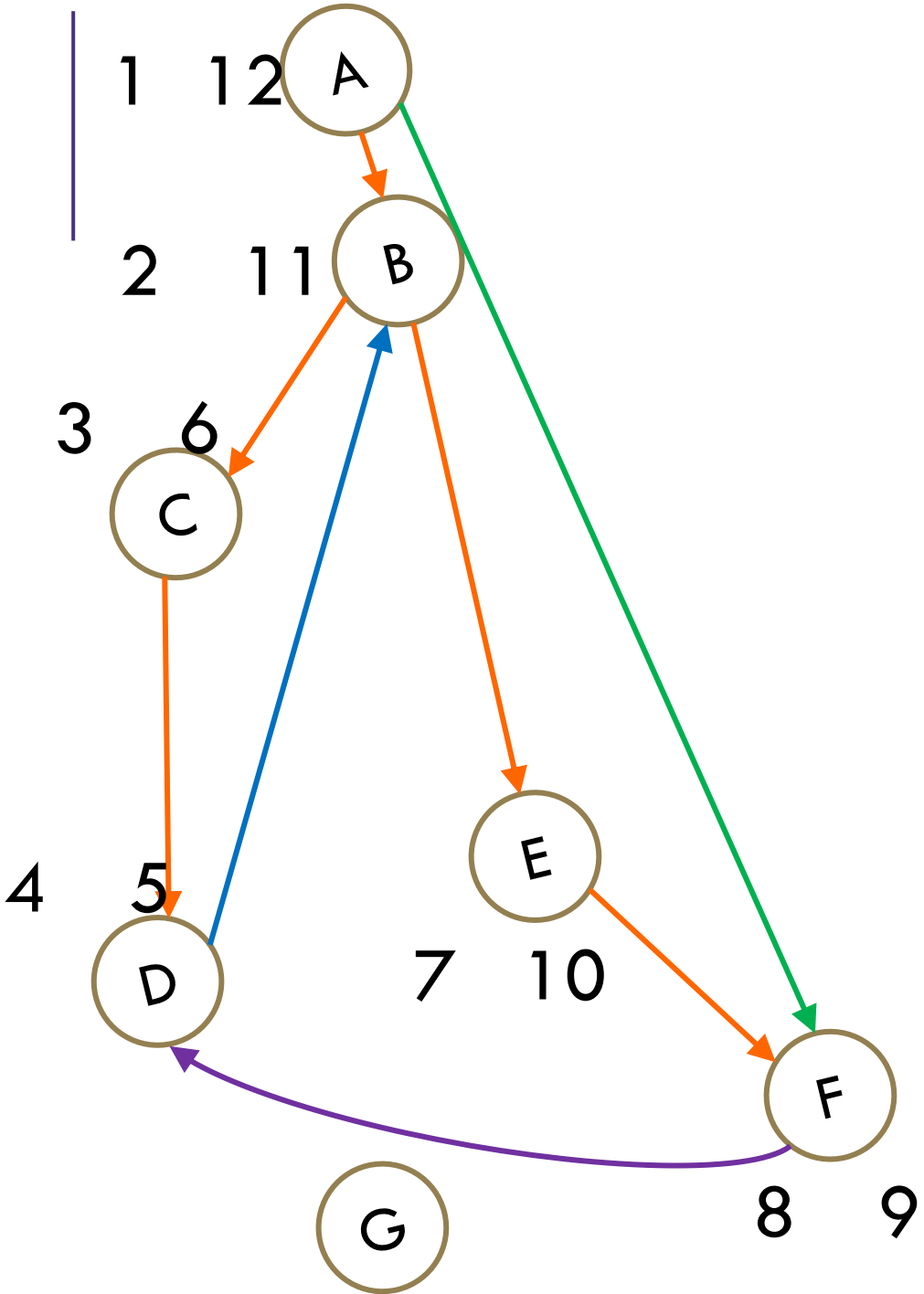
Vertex: E
Last edge used: (E,F)

Vertex: B
Last edge used: (B,E)

Vertex: A
Last edge used: (A,F)




```
DFS(u)
        Mark u as "seen"
        u.start = counter++
        For each edge (u,v) //leaving u
              If v is not "seen"
                    DFS(v)
              End If
        End For
        u.end = counter++
```



The orange edges (the ones where we discovered a new vertex) form a tree!*
We call them **tree edges.**

That blue edge went from a descendent to an ancestor
B was still on the stack when we found (B,D).
We call them **back edges.**

The green edge went from an ancestor to a descendant
F was put on and come off the stack between putting A on the stack and finding (A,F)
We call them **forward edges.**

The purple edge went…some other way.
D had been on and come off the stack before we found F or (F,D)
We call those **cross edges.**

*Conditions apply. Sometimes the graph is a forest. But we call them tree edges no matter what.

Edge Classification (for DFS on directed graphs)

Edge type	Definition	When is (u, v) that edge type?
Tree	Edges forming the DFS tree (or forest).	v was not seen before we processed (u, v).
Forward	From ancestor to descendant in tree.	u and v have been seen, and `u.start < v.start < v.end < u.end`
Back	From descendant to ancestor in tree.	u and v have been seen, and `v.start < u.start < u.end < v.end`
Cross	Edges going between vertices without an ancestor relationship.	u and v have not been seen, and `v.start < v.end < u.start < u.end`

The third column doesn't look like it encompasses all possibilities.

It does – the fact that we're using a stack limits the possibilities:

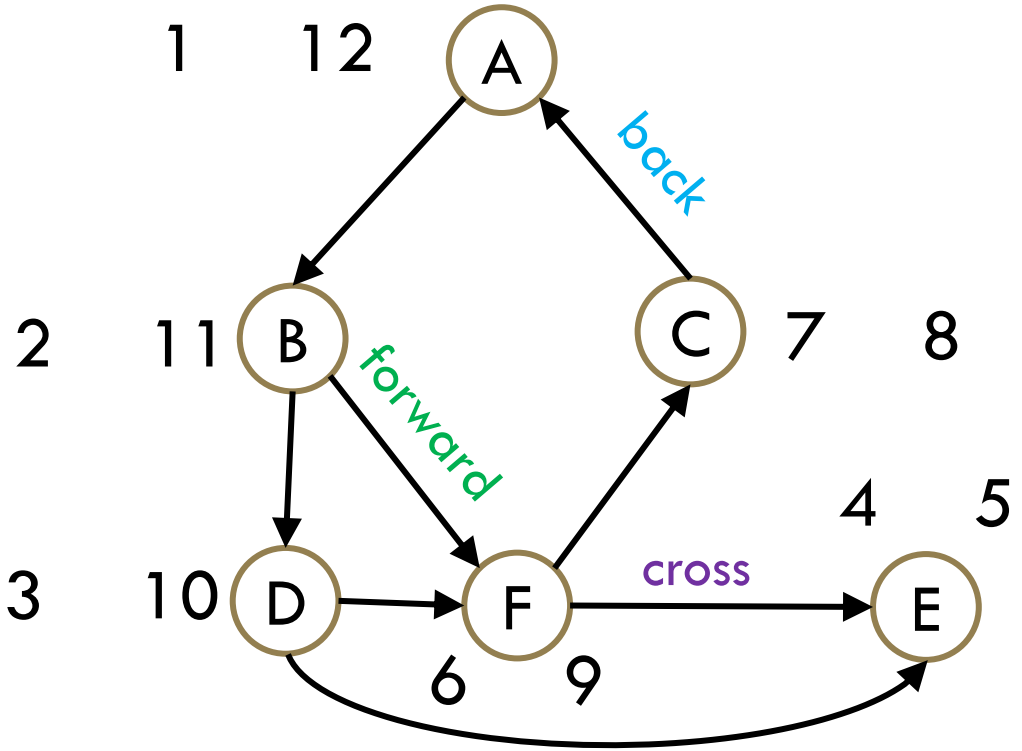
e.g. `u.start < v.start < u.end < v.end` is impossible.

And the rules of the algorithm eliminate some other possibilities.

Try it Yourselves!

```
DFSWrapper(G)
     counter = 0
     For each vertex u of G
          If u is not "seen"
               DFS(u)
          End If
     End For

DFS(u)
     Mark u as "seen"
     u.start = counter++
     For each edge (u,v) //leaving u
          If v is not "seen"
               DFS(v)
          End If
     End For
     u.end = counter++
```



Actually Using DFS

Here's a claim that will let us use DFS for something!

Back Edge Characterization
DFS run on a directed graph has a back edge if and only if it has a cycle.

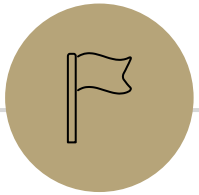
Not responsible for the details, just notice the similarity to the type of proof we did before (differentiate types of edges, use them to find information)
More examples in any of the books on the resources tab. (also this proof in last quarter's slides).

BFS/DFS caveats and cautions

Edge classifications are different for directed graphs and undirected graphs.

DFS in undirected graphs don’t have cross edges.

BFS in directed graphs can have edges skipping levels (only as back edges, skipping levels up though!)



Graph Search Takeaways

Your Takeaways

When searching through a graph, order matters!

BFS and DFS do different things!

BFS/DFS algorithms usually keep track of extra information/calculate something at each vertex/use edge classification to solve the problem.

A few extra bells and whistles in the code, but usually little more.

DFS can solve a wide-variety of problems, but the algorithms tend to be subtle.

BFS a lot more intuitive, start there if you can.

BE CAREFUL with directed/undirected graphs. The algorithms aren't always easy to convert.

Summary – Graph Search Applications

BFS

Shortest Paths (unweighted) graphs)

DFS

Cycle detection (directed graphs)

Topological sort

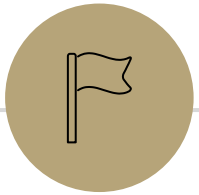
Strongly connected components

EITHER

2-coloring

Connected components (undirected)

Usually use BFS – easier to understand.



Solving new problems: Graph Modeling

Problem 1: Ordering Dependencies

Given a directed graph G, where we have an edge from u to v if u must happen before v.

We can only do things one at a time, can we find an order that **respects dependencies**?

Topological Sort (aka Topological Ordering)

Given: a directed graph G

Find: an ordering of the vertices so all edges go from left to right.

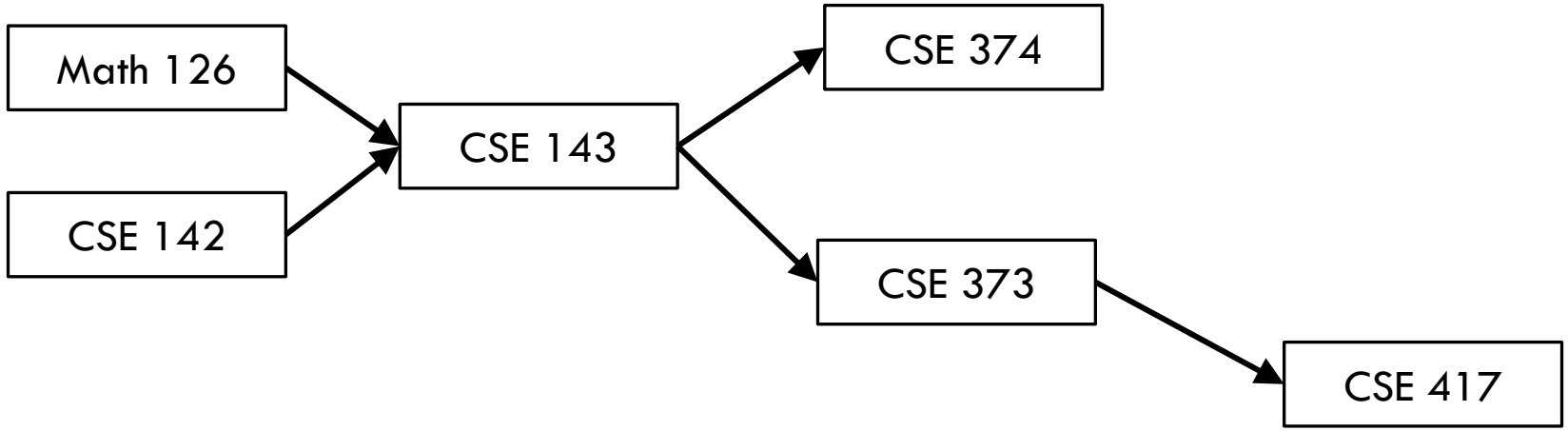
Uses:

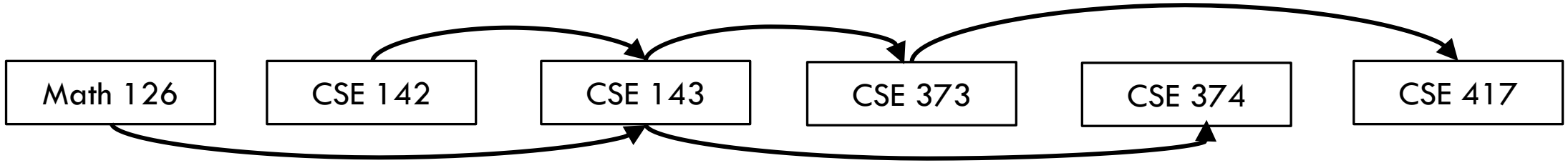
Compiling multiple files

Graduating

Topological Ordering

A course prerequisite chart and a possible topological ordering.



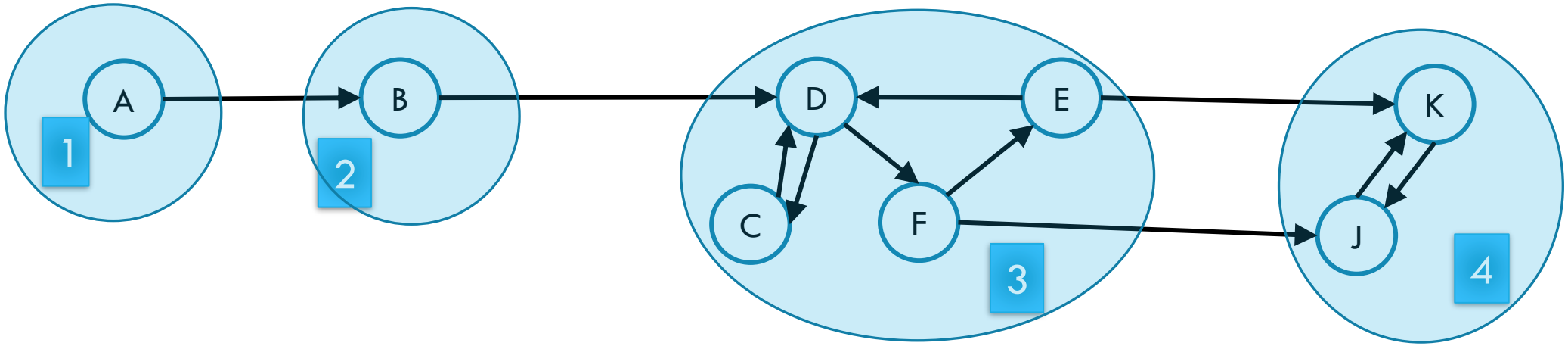


Problem 2

Given a graph, find its strongly connected components

Strongly Connected Component

A set of vertices C such that every pair of vertices in C is connected via some path **in both directions**, and there is no other vertex which is connected to every vertex of C in both directions.



How do these work?

A couple of different ways to use DFS to find strongly connected components.

Wikipedia has the details.

High level: need to keep track of "highest point" in DFS tree you can reach back up to.

You can use this algorithm as a library function!

We also listed all the ones from 332 on this webpage.

Topological sort

You saw an algorithm in 332

Important thing: both run in $\Theta(m + n)$ time.

Designing New Algorithms

When you need to design a new algorithm on graphs, whatever you do is probably going to take at least $\Omega(m + n)$ time.

So you can run any $O(m + n)$ algorithm as "preprocessing"

Finding connected components (undirected graphs)

Finding SCCs (directed graphs)

Do a topological sort (DAGs)

Designing New Algorithms (combining)

Finding SCCs and topological sort go well together:

From a graph G you can define the "meta-graph" G^{SCC}
(aka "condensation", aka "graph of SCCs")

G^{SCC} has a vertex for every SCC of G

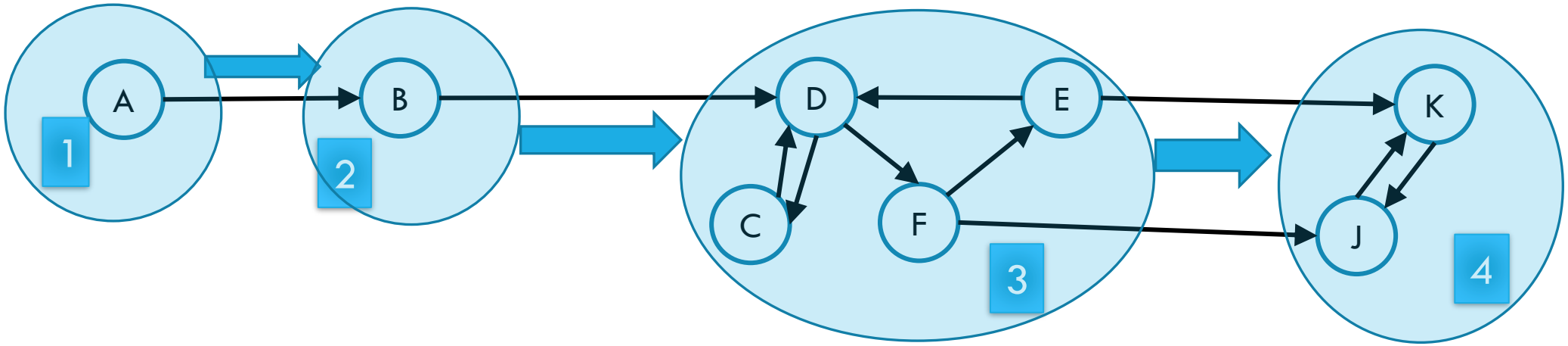
There's an edge from u to v in G^{SCC} if and only if there's an edge in G from a vertex in u to a vertex in v.

Why Find SCCs?

Let's build a new graph out of them! Call it G^{SCC}

Have a vertex for each of the strongly connected components

Add an edge from component 1 to component 2 if there is an edge from a vertex inside 1 to one inside 2.



Designing New Graph Algorithms

Not a common task – most graph problems have been asked before.

When you need to do it, Robbie recommends:

Start with a simpler case (topo-sorted DAG or [strongly] connected graph).

HW problem walks you through designing an algorithm by:

1. Figuring out what you'd do if the graph is strongly connected
2. Figuring out what you'd do if the graph is a topologically ordered DAG
3. Stitching together those two ideas (using G^{scc}).

Problem Solving Suggestions

Read the problem carefully.

Are there any technical terms in the question? Any formulas?

What kind of object will you get as input? What type is your output?

Do you understand it? Write sample inputs and outputs

We'll often give you samples, but it helps to add your own.

Now start thinking about solutions

On those examples, how would you get the solution?

Does this remind you of any algorithms from class?

Can you think of a new idea?

It's ok to start with slow solutions and try to speed them up!

Try the graph modeling process.

Graph Modeling

But…Most of the time you don't need a new graph algorithm.

What you need is to figure out what graph to make and which graph algorithm to run.

"Graph modeling"

Going from word problem to graph algorithm.

Often finding a clever way to turn your requirements into graph features.

Mix of "standard bag of tricks" and new creativity.

Graph Modeling Process

1. What are your fundamental objects?

Those will probably become your vertices.

2. How are those objects related?

Represent those relationships with edges.

3. How is what I'm looking for encoded in the graph?

Do I need a path from s to t? The shortest path from s to t? A minimum spanning tree? Something else?

4. Do I know how to find what I'm looking for?

Then run that algorithm/combination of algorithms

Otherwise go back to step 1 and try again.

Scenario #1

You've made a new social networking app, Convrs. Users on Convrs can have "asymmetric" following (I can follow you, without you following me). You decide to allow people to form multi-user direct messages, but only if people are probably in similar social circles (to avoid spamming).

You'll allow a messaging channel to form only if for every pair of users a,b in the channel: a must follow b or follow someone who follows b or follow someone who follows someone who follows b, or …
And the same for b to a.

You'd like to be able to quickly check for any new proposed channel whether it meets this condition.

What are the vertices?

What are the edges?

What are we looking for?

What do we run?

Scenario #1 (answer)

You've made a new social networking app, Convrs. Users on Convrs can have "asymmetric" following (I can follow you, without you following me). You decide to allow people to form multi-user direct messages, but only if people are probably in similar social circles (to avoid spamming).

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And the same for b to a.

You'd like to be able to quickly check for any new proposed channel whether it meets this condition.

What are the vertices?

Users

What are the edges?

Directed – from u to v if u follows v

What are we looking for?

If everyone in the channel is in the same SCC.

What do we run?

Find SCCs, to test a new channel, make sure all are in same component.

Scenario #2

Sports fans often use the “transitive law” to predict sports outcomes -- In general, if you think A is better than B, and B is also better than C, then you expect that A is better than C.

Teams don’t all play each other – from data of games that have been played, determine if the “transitive law” is realistic, or misleading about at least one outcome.

What are the vertices?

What are the edges?

What are we looking for?

What do we run?

Scenario #2 (answer)

Sports fans often use the "transitive law" to predict sports outcomes -- . In general, if you think A is better than B, and B is also better than C, then you expect that A is better than C.

Teams don't all play each other – from data of games that have been played, determine if the "transitive law" is realistic, or misleading about at least one outcome.

What are the vertices?

Teams

What are the edges?

Directed – Edge from u to v if u beat v.

What are we looking for?

A cycle would say it's not realistic.
OR a topological sort would say it is.

What do we run?

Cycle-detection DFS.
a topological sort algorithm (with error detection)

Scenario #3

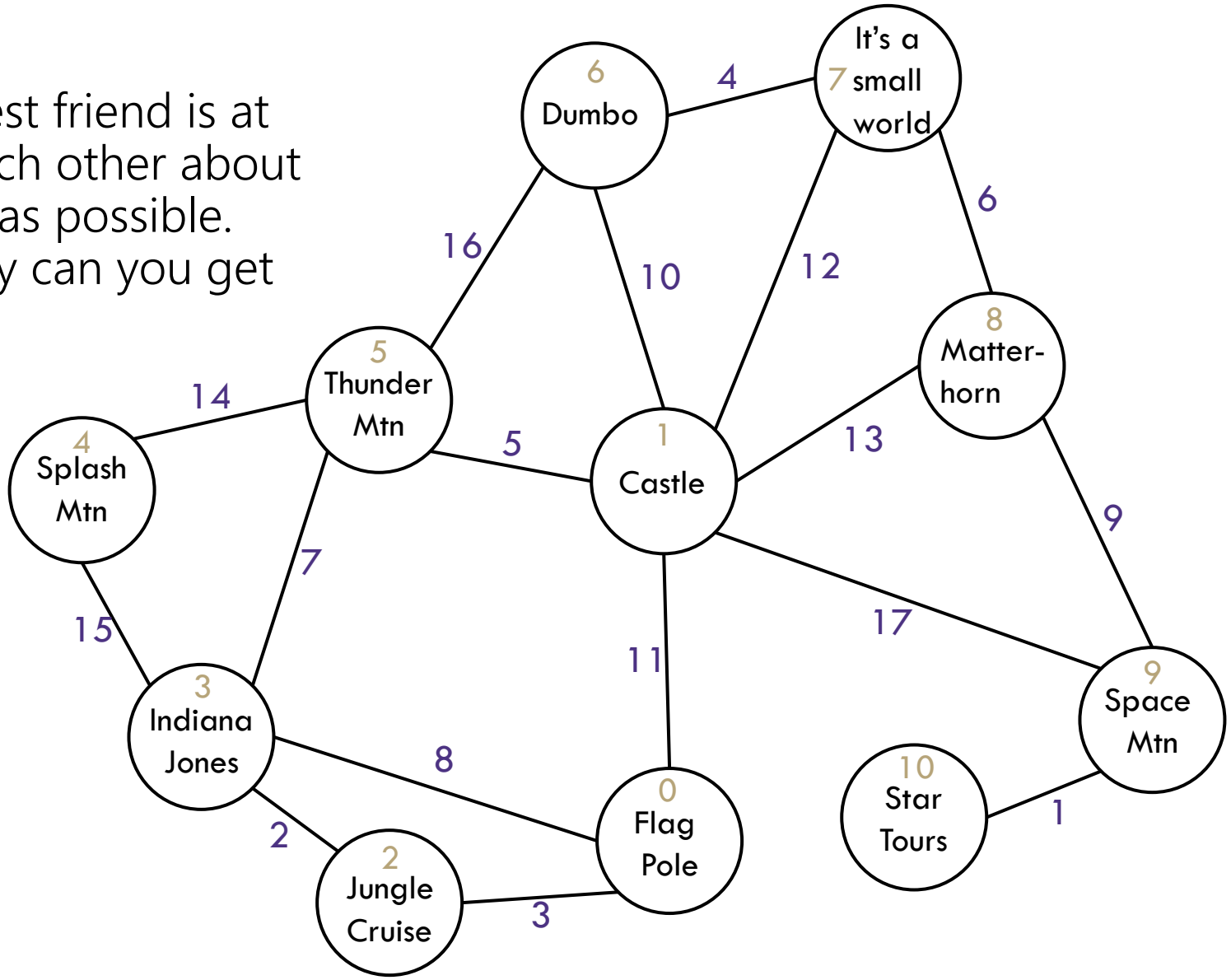
You are at Splash Mountain. Your best friend is at Space Mountain. You have to tell each other about your experiences in person as soon as possible. Where do you meet and how quickly can you get there?

What are the vertices?

What are the edges?

What are we looking for?

What do we run?



Scenario #3 (answer)

You are at Splash Mountain. Your best friend is at Space Mountain. You have to tell each other about your experiences in person as soon as possible. Where do you meet and how quickly can you get there?

What are the vertices?

Rides

What are the edges?

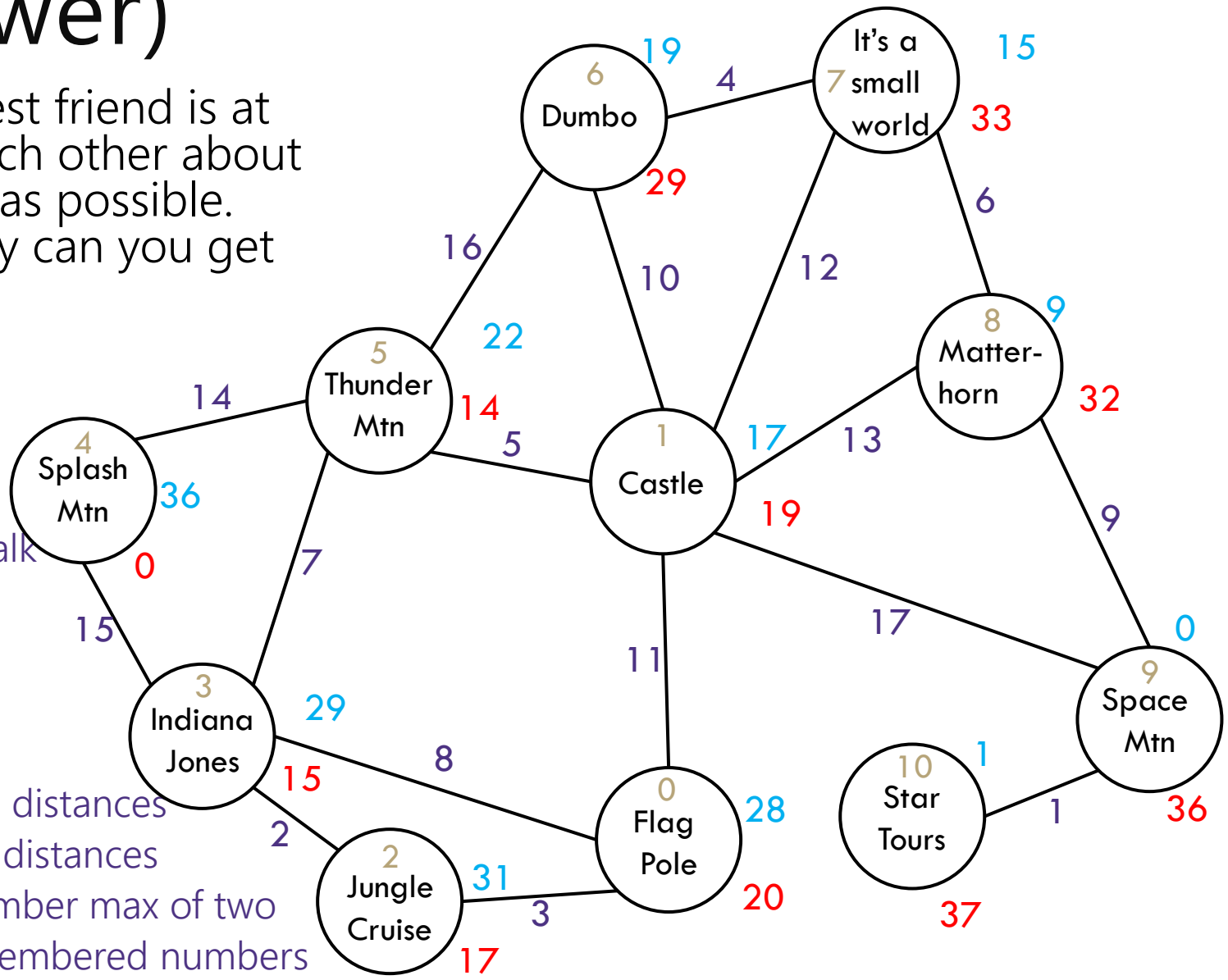
Walkways with how long it would take to walk

What are we looking for?

- The "midpoint"

What do we run?

- Run Dijkstra's from Splash Mountain, store distances
- Run Dijkstra's from Space Mountain, store distances
- Iterate over vertices, for each vertex remember max of two
- Iterate over vertices, find minimum of remembered numbers



Scenario #4

You're a Disneyland employee, working the front of the Splash Mountain line. Suddenly, the crowd-control gates fall over and the line degrades into an unordered mass of people.

Sometimes you can tell who was in line before who; for other groups you aren't quite sure. You need to restore the line, while ensuring if you **knew** A came before B before the incident, they will still be in the right order afterward.

What are the vertices?

People

What are the edges?

Edges are directed, have an edge from X to Y if you know X came before Y.

What are we looking for?

- A total ordering consistent with all the ordering we do know.

What do we run?

- Topological Sort!