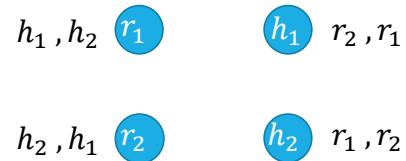


## Multiple Stable Matchings

Suppose we take our algorithm and let the horses do the "proposing" instead.



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```
//preprocess, Inv[i][j] answers the question "horse i, what position in your list is rider j?"
for(int k=0; k<n; k++){
    int riderNum = horse[i][k];
    Inv[i][riderNum]=k;
}
```

Initially all  $r$  in  $R$  and  $h$  in  $H$  are free

while there is a free  $r$

    Let  $h$  be highest on  $r$ 's list that  $r$  has not proposed to

    if  $h$  is free

        match ( $r, h$ )

    else //  $h$  is not free

        Let  $r'$  be the current match of  $h$ .

        if  $Inv[h][r] < Inv[h][r']$  //  $h$  prefers  $r$  to  $r'$ .

            unmatch ( $r', h$ )

            match ( $r, h$ )

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## Polynomial vs. Exponential

If you have an algorithm that takes exactly  $f(n)$  microseconds, how large of an  $n$  can you handle in the given time?

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$	$2^{10^6}$	$2^{6 \cdot 10^7}$	$2^{36 \cdot 10^8}$	$2^{864 \cdot 10^8}$	$2^{25920 \cdot 10^8}$	$2^{315360 \cdot 10^8}$	$2^{31556736 \cdot 10^8}$
$\sqrt{n}$	$10^{12}$	$36 \cdot 10^{14}$	$1296 \cdot 10^{16}$	$746496 \cdot 10^{16}$	$6718464 \cdot 10^{18}$	$994519296 \cdot 10^{18}$	$995827586973696 \cdot 10^{16}$
$n$	$10^6$	$6 \cdot 10^7$	$36 \cdot 10^8$	$864 \cdot 10^8$	$2592 \cdot 10^9$	$31536 \cdot 10^9$	$31556736 \cdot 10^8$
$n \lg n$	62746	2801417	133378058	2755147513	71870856404	797633893349	68654697441062
$n^2$	1000	7745	60000	293938	1609968	5615692	56175382
$n^3$	100	391	1532	4420	13736	31593	146677
$2^n$	19	25	31	36	41	44	51
$n!$	9	11	12	13	15	16	17

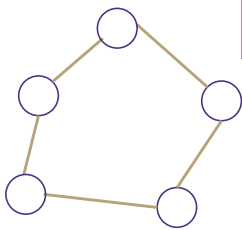
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## A detailed application

### Bipartite (also called "2-colorable")

A graph is bipartite (also called 2-colorable) if the vertex set can be divided into two sets  $V_1, V_2$  such that the only edges go between  $V_1$  and  $V_2$ .

Called "2-colorable" because you can "color"  $V_1$  red and  $V_2$  blue, and no edge connects vertices of the same color.



If a graph contains an odd cycle, then it is not bipartite.

Try the example on the right, then proving the general theorem in the light purple box.

Help Robbie figure out how long to make the explanation  
[Pollev.com/robbie](https://pollev.com/robbie)

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