Stable Matching, More Formally

Perfect matching:

- Each rider is paired with exactly one horse.
- Each horse is paired with exactly one rider.

Stability: no ability to exchange

an unmatched pair r-h is blocking if they both prefer each other to current matches.

Stable matching: perfect matching with no blocking pairs.

Stable Matching Problem

Given: the preference lists of n riders and n horses.
Find: a stable matching.

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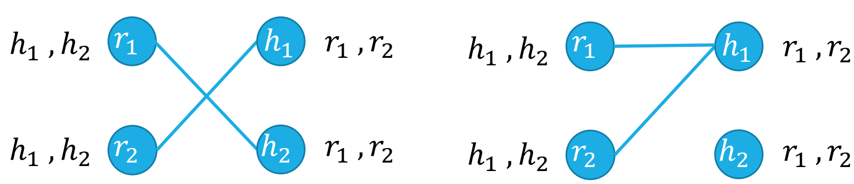
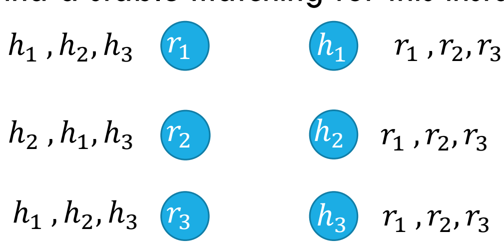
Try it!

Why are these not stable matchings?



Find a stable matching for this instance.



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Gale-Shapley Algorithm

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Initially all r in R and h in H are free
while there is a free r
     Let h be highest on rs list that r has not proposed to
     if h is free
         match (r,h)
     else //h is not free
         Let r′ be the current match of h.
         if h prefers r to r′
               unmatch (r′,h)
               match (r,h)
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Claim 1: If r proposed to the last horse on their list, then all the horses are matched. (2)

Try to prove this claim, i.e. clearly explain why it is true. You might want some of these observations:

- Observation A: r's proposals get worse (for r).
- Observation B: Once h is matched, h never becomes free again.
- Observation C: h's partners cannot get worse (for h).

Hint: r must have been rejected a lot – what does that mean?

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