# CSE 421 Spring 2025: Set 9 (just for fun!)

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Due date: Thursday June 12th, 2025 11:59pm

**Problem 1 (0 bonus points).** Recall coNP to be class of decision problems whose complement is in NP. Equivalently its the set of problems $X$ such that $x \in X$ iff there exists a certifier $\mathcal{V}$ such that $\mathcal{V}(x, \pi) = 1$ for all $\pi$. Prove that NP $\neq$ coNP implies $P \neq$ NP.

**Problem 2 (0 bonus points).** A one-way function family is a set of functions $f_n : \{0,1\}^n \to \{0,1\}^m$ such that

1. there exists some $n^c$-time algorithm $\mathcal{A}$ such that $\mathcal{A}(x) = f_n(x)$ where $n = |x|$.
2. for all poly-time computable algorithms $\mathcal{B}$, there exists a $y$ such that $f(\mathcal{B}(y)) \neq y$ — i.e., the inverse is not efficiently computable.

Show that the existence of one-way functions implies P $\neq$ NP.

**Problem 3 (0 bonus points).** Write down the description of an algorithm $\mathcal{A}$ such that

1. if input the description $\langle G \rangle$ of a graph, it necessarily halts and outputs if the graph is 3-colorable.
2. if P = NP, then this algorithm $\mathcal{A}$ must halt in polynomial-time $|G|^c$ for some constant $c > 0$.

Hint: There is an enumeration of all possible algorithms as they can be described by Turing machines.