Lecture 17

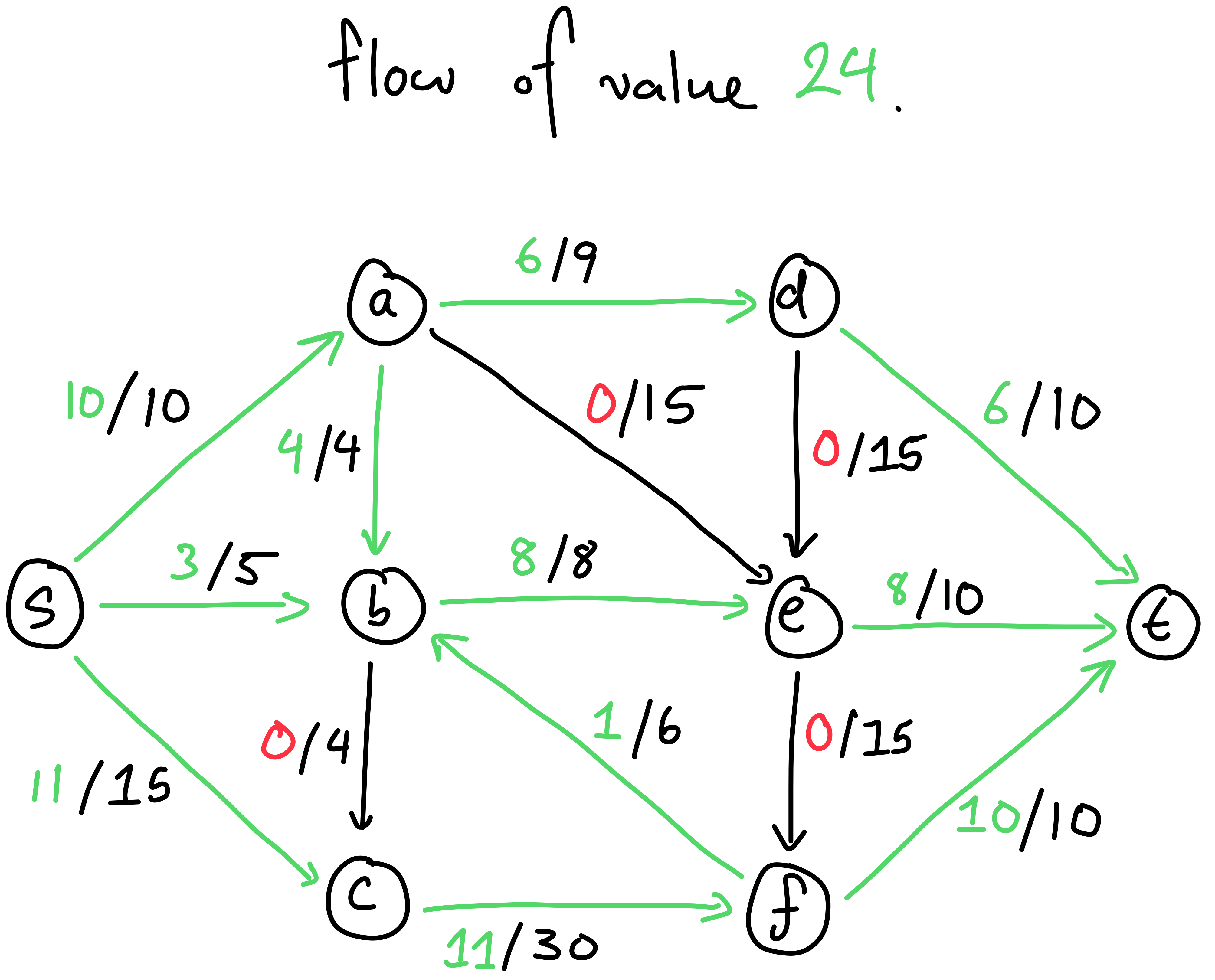
Efficient Maximum Flow and applications

Chinmay Nirkhe | CSE 421 Spring 2025

Previously in CSE 421…

The maximum flow problem

- **Input:** a flow network (G, c, s, t)
- **Output:** a s-t flow of maximum value



Today

Ford-Fulkerson always finds a max flow

- **Theorem:** When capacities are positive integers, Ford-Fulkerson always terminates and outputs a max-flow.

- **Observation:** Ford-Fulkerson only terminates if there is no path $s \rightsquigarrow t$ in the residual graph G_f.

- Therefore, it suffices to show that a flow f is maximal iff there is no no path $s \rightsquigarrow t$ in the residual graph G_f.

Let's prove this!

The max flow/min cut theorem

- **Max flow/min cut theorem:** Let f be a flow in a network (G, s, t, c). The following statements are equivalent!
  - (1) There exists a s-t cut (S, T) such that $v(f) = c(S, T)$.
  - (2) f is a max flow.
  - (3) There is no augmentation path $s \rightsquigarrow t$ in G_f.
- We will prove that (1) $\implies$ (2), (2) $\implies$ (3), and (3) $\implies$ (1).

The max flow/min cut theorem

(1) $\implies$ (2)

- (1) There exists a s-t cut (S, T) such that $v(f) = c(S, T)$.
- (2) f is a max flow.
- **Proof**:
  - We know that $v(f) \leq c(S, T)$ for any s-t cut [Weak duality].
  - So if $v(f) = c(S, T)$, then there cannot be any flow f' s.t. $v(f') > v(f)$.
  - So f must be maximal.

The max flow/min cut theorem

(2) $\implies$ (3)

- (2) f is a max flow.
- (3) There is no augmentation path $s \rightsquigarrow t$ in G_f.

- **Proof:** By contrapositive.

The max flow/min cut theorem

$\neg$ (3) $\implies \neg$ (2)

- $\neg$ (2) f is **not** a max flow.
- $\neg$ (3) There **is** a augmentation path $s \rightsquigarrow t$ in G_f.

- **Proof**:
  - Let f_{aug} be the augmentation path.
  - We saw last lecture that $f + f_{\text{aug}}$ is a flow in G. And $v(f + f_{\text{aug}}) > v(f)$.

The max flow/min cut theorem

(3) $\implies$ (1)

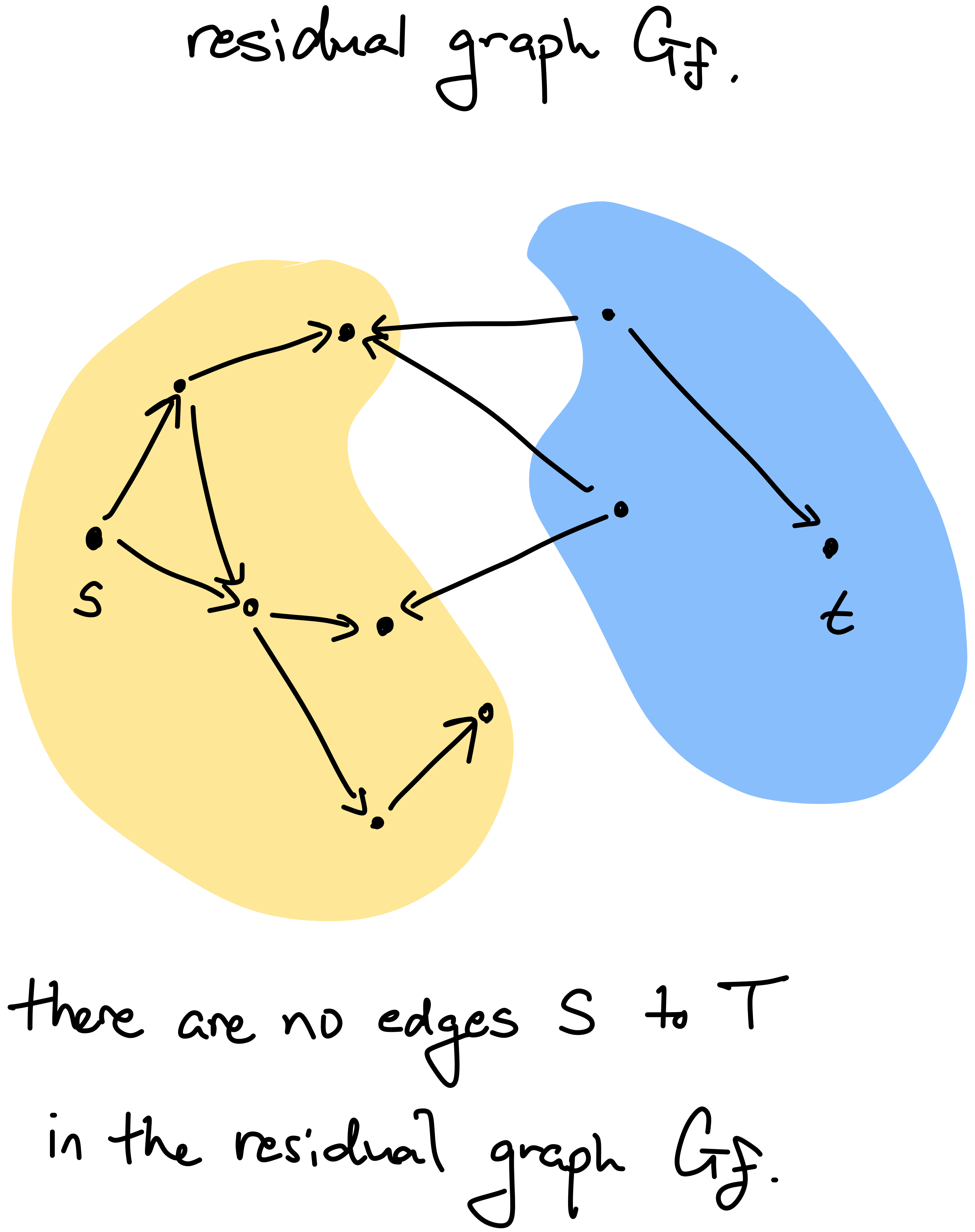
- (3) There is no augmentation path $s \rightsquigarrow t$ in G_f.
- (1) There exists a s-t cut (S, T) such that $v(f) = c(S, T)$.

- **Proof:** This is a lengthy proof! It will take us a few slides. Key ideas:
  - We will need to find the s-t cut (S, T). It should be based on the aug. path.
  - Then we will use that $v(f) = f^{\text{out}}(S) - f^{\text{in}}(S)$ to prove that $v(f) = c(S, T)$.

The max flow/min cut theorem

(3) $\implies$ (1)

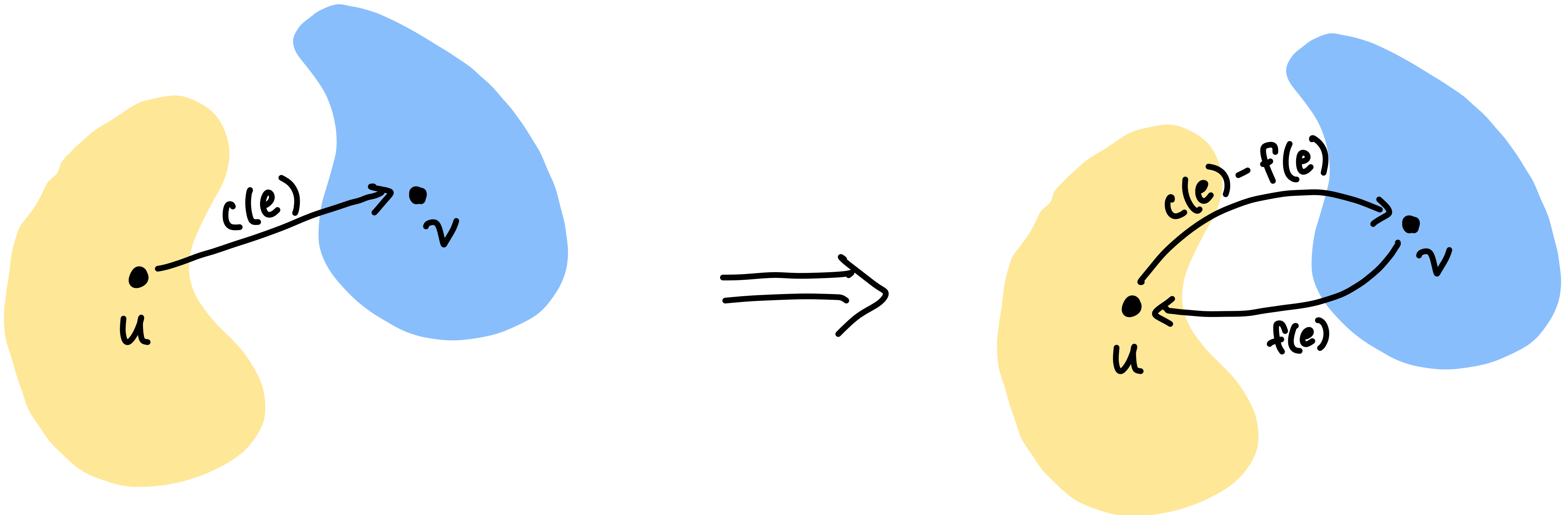
- **Proof**:
  - Let f be a flow such that there are no augmenting paths in G_f.
  - Let S be the set of vertices reachable from s.
    - Since there are no paths, $t \notin S$.
    - Let $T = V \backslash S$ and this defines a s-t cut.



The max flow/min cut theorem

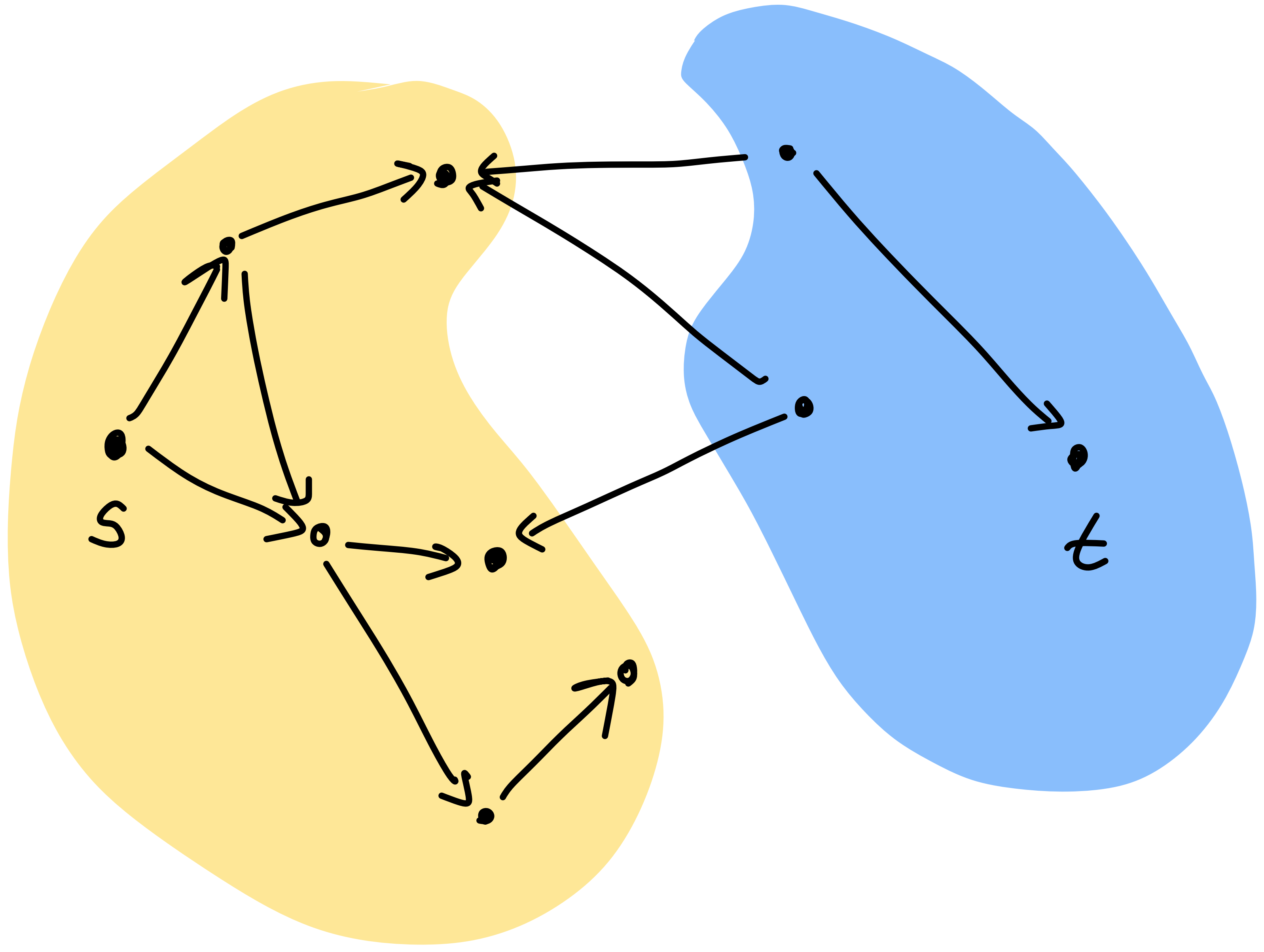
(3) $\implies$ (1)

- **Proof**:
  - What does it mean for there to be no edges S to T in the residual graph G_f?
  - For any edge $e = (u \to v) \in G$ from S to T,



edge in the graph G   edges in residual graph G_f

residual graph G_f.

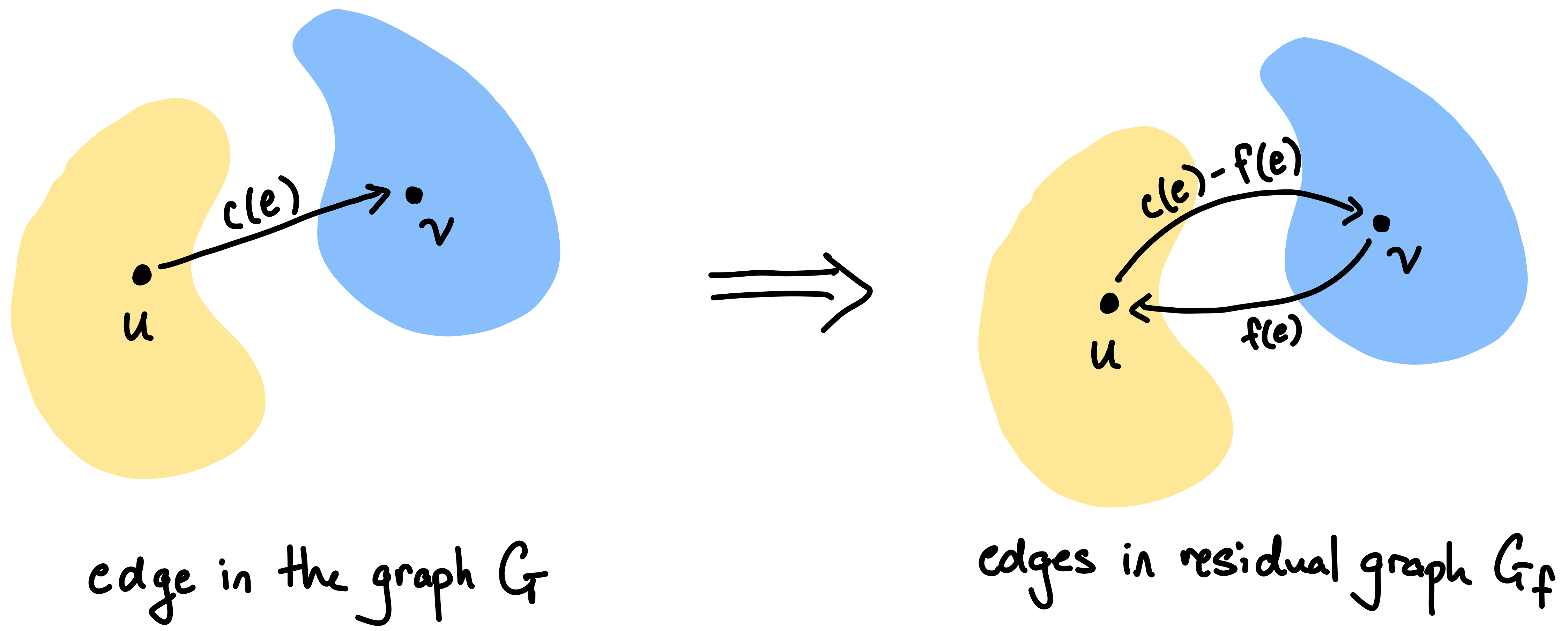


there are no edges S to T in the residual graph G_f.

The max flow/min cut theorem

(3) $\implies$ (1)

- **Proof:**
  - What does it mean for there to be no edges S to T in the residual graph G_f?
  - For any edge $e = (u \to v) \in G$ from S to T,

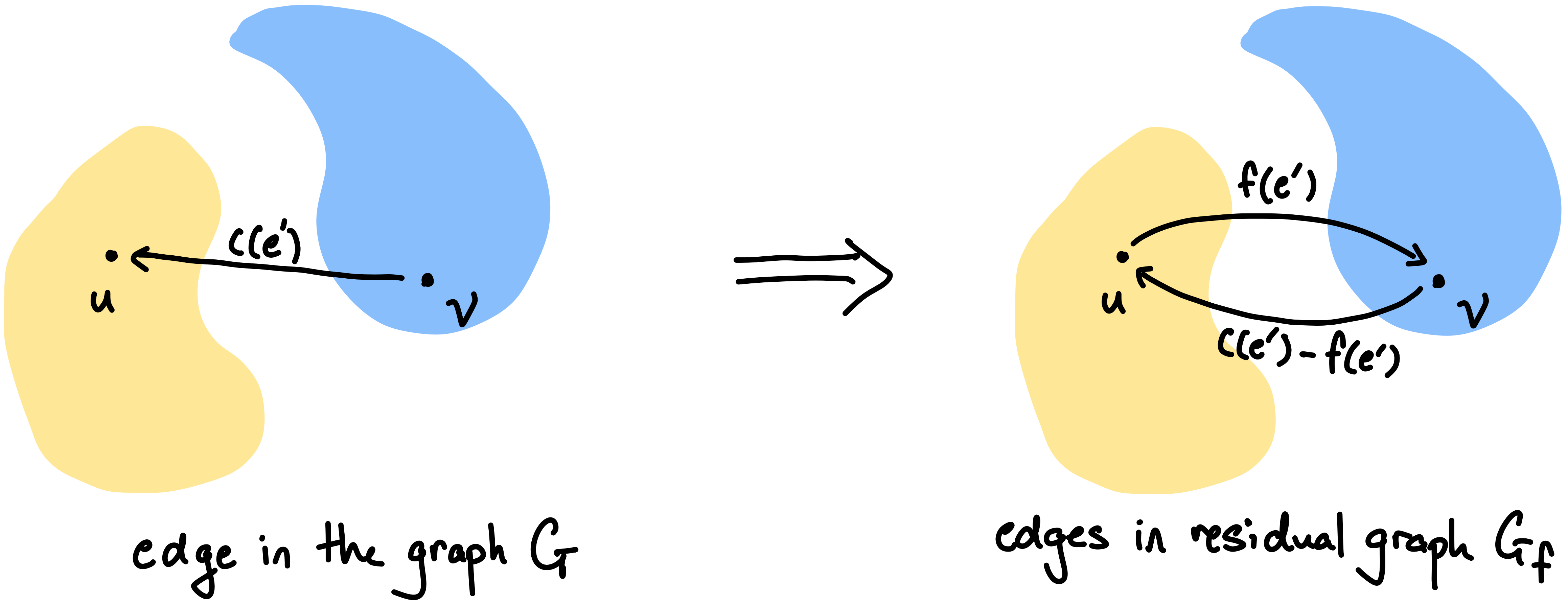


Therefore, $c(e) = f(e)$ for all edges $u \to v$ from S to T.

The max flow/min cut theorem

(3) $\Longrightarrow$ (1)

- **Proof:**
  - What does it mean for there to be no edges S to T in the residual graph G_f?
  - For any edge $e' = (v \to u) \in G$ from T to S,

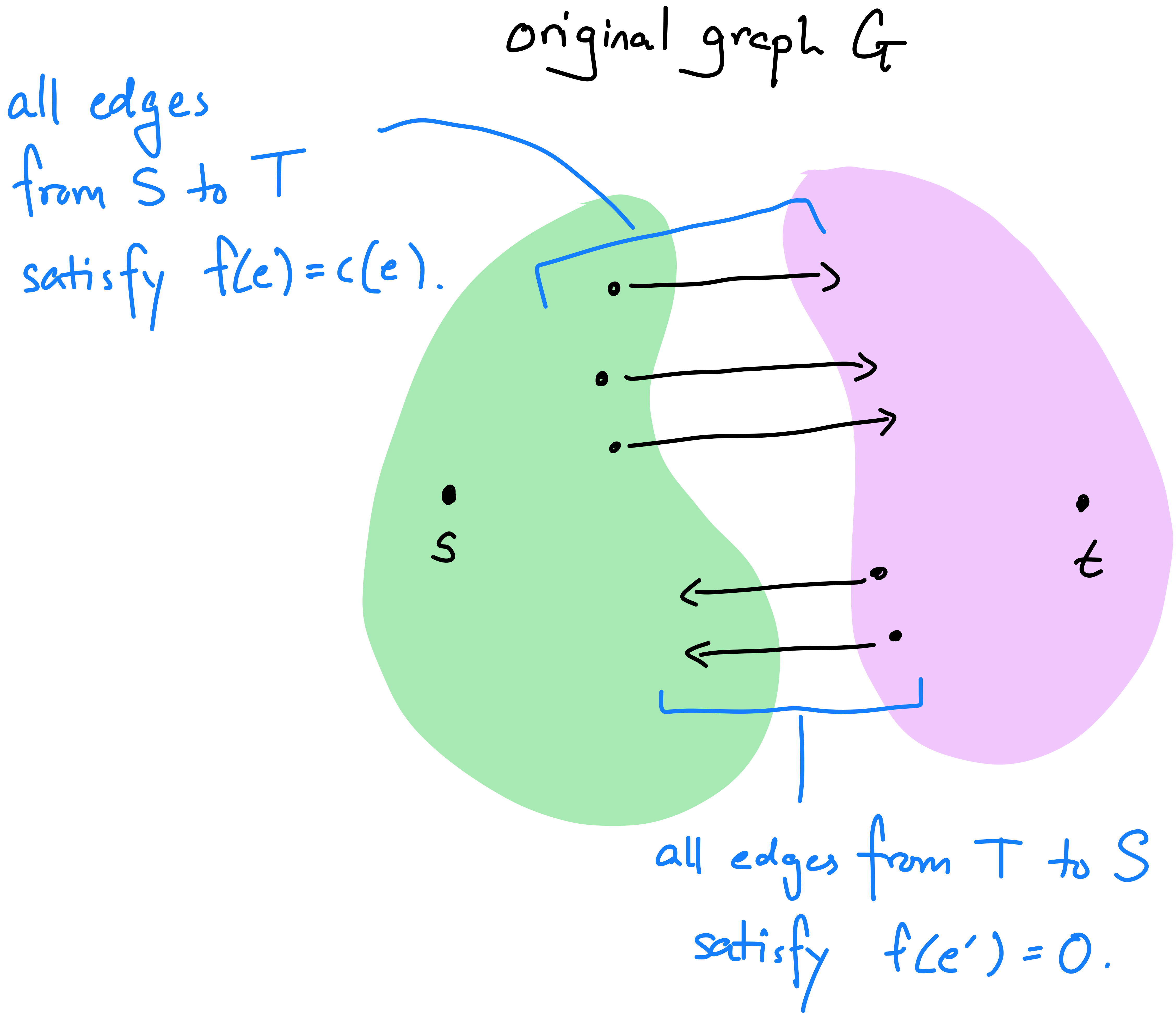


Therefore, $f(e') = 0$ for all edges $v \to u$ from T to S.

The max flow/min cut theorem

(3) $\Longrightarrow$ (1)

- **Proof**:
  - Edges from S to T are *saturated* with flow.
  - Edges from T to S have no flow.
  - $v(f) = f^{\text{out}}(S) - f^{\text{in}}(S)$
  - $= \sum_{e \text{ from } S \text{ to } T} f(e) - \sum_{e' \text{ from } T \text{ to } S} f(e')$
  - $= \underbrace{\sum_{e \text{ from } S \text{ to } T} c(e)}_{C(S,T)} - \underbrace{\sum_{e' \text{ from } T \text{ to } S} 0}_{0}$
  - $= C(S,T)$.



The max flow/min cut theorem

(3) $\implies$ (1)

- (3) There is no augmentation path $s \rightsquigarrow t$ in G_f.
- (1) There exists a s-t cut (S, T) such that $v(f) = c(S, T)$.

- **Proof:** This is a lengthy proof! It will take us a few slides. Key ideas:
  - We will need to find the s-t cut (S, T). It should be based on the aug. path.
  - Then we will use that $v(f) = f^{\text{out}}(S) - f^{\text{in}}(S)$ to prove that $v(f) = c(S, T)$.

The max flow/min cut theorem

- **Max flow/min cut theorem:** Let f be a flow in a network (G, c, s, t). The following statements are equivalent!
  - (1) There exists a s-t cut (S, T) such that $v(f) = c(S, T)$.
  - (2) f is a max flow.
  - (3) There is no augmentation path $s \rightsquigarrow t$ in G_f.
- **Corollary:** The value of the max flow equals the value of the min cut!

Returning to Ford-Fulkerson

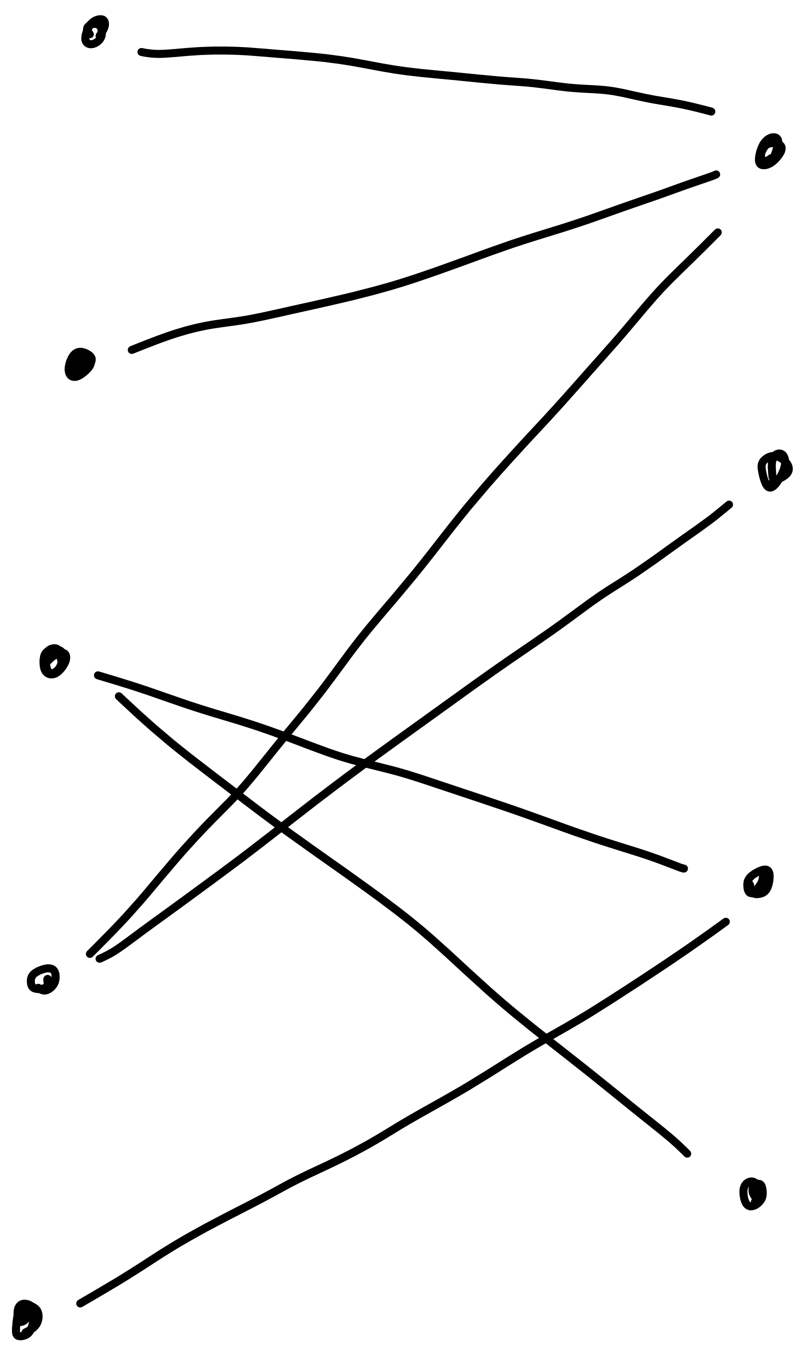
- Ford-Fulkerson is a greedy algorithm which calculates the max flow by incrementally increasing the flow.
- Max flow/min cut theorem proves that Ford-Fulkerson only terminates when the max flow is achieved.
- If the capacities are integer, Ford-Fulkerson will increase the flow by at least 1 per iteration.
- Yields a runtime of $O(mC)$ where C is the sum of capacities of edges leaving s.
- Runtime can be exponential time in input length for large C as capacities are expressed in binary in the input.
- But when $C = \text{poly}(n)$, then algorithm can be very efficient.

Integral max flow

- **Theorem:** Consider a graph network (G, s, t, c) where $c : E \to \mathbb{Z}_{\geq 0}$. Then, there exists a max flow which assigns an integer flow to every edge.
- **Proof:**
  - Ford-Fulkerson will calculate the max flow.
  - Ford-Fulkerson only increases the flow by integer quantities starting from 0.
  - Therefore, there exists a max flow that has integer flow.

Application: Bipartite matching

- **Input:** A bipartite graph $(V = L \sqcup R, E)$
- **Output:** A maximal collection of edges that don't share any vertices.

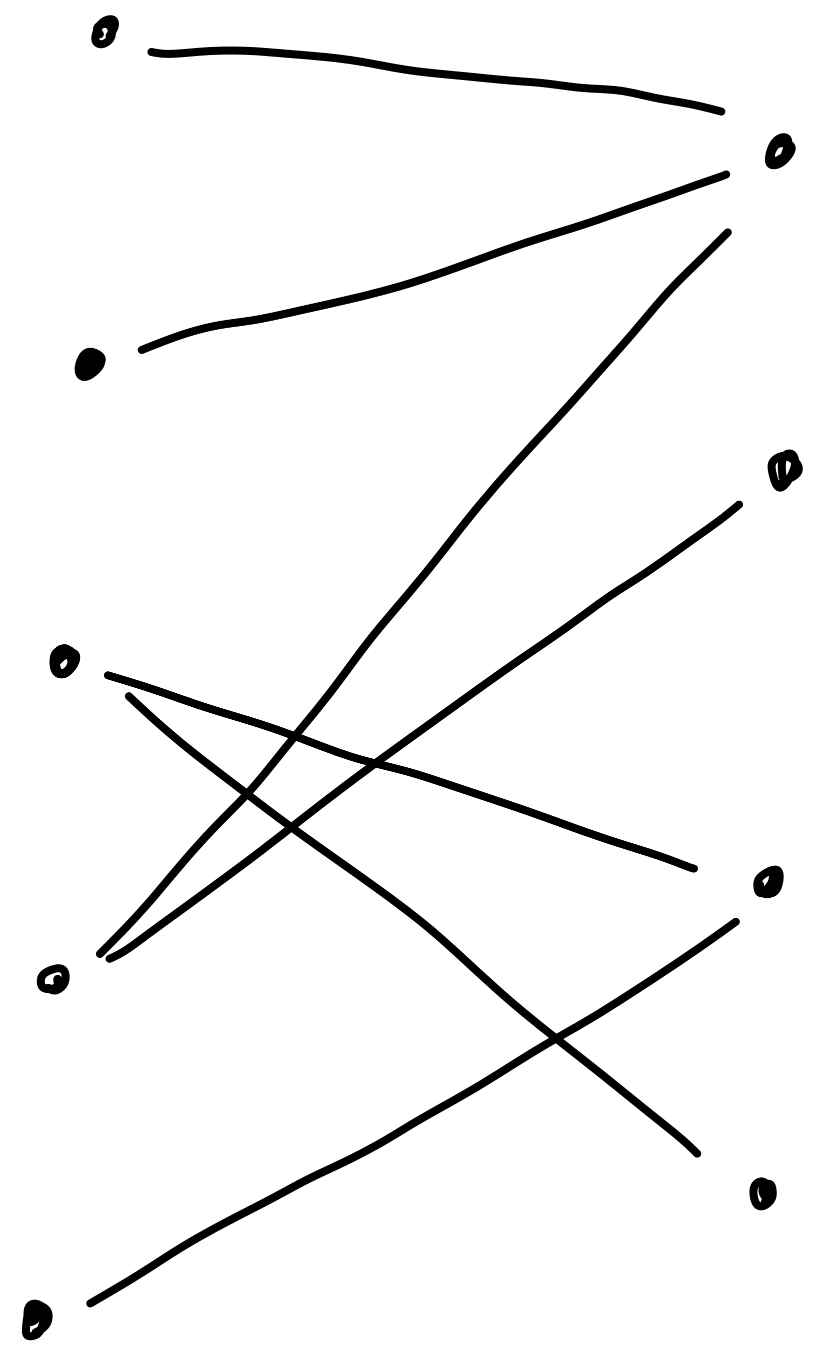


- We saw this problem earlier in the course, but didn't come up with an algorithm.
- We will see that there is an algorithm based on Ford-Fulkerson.

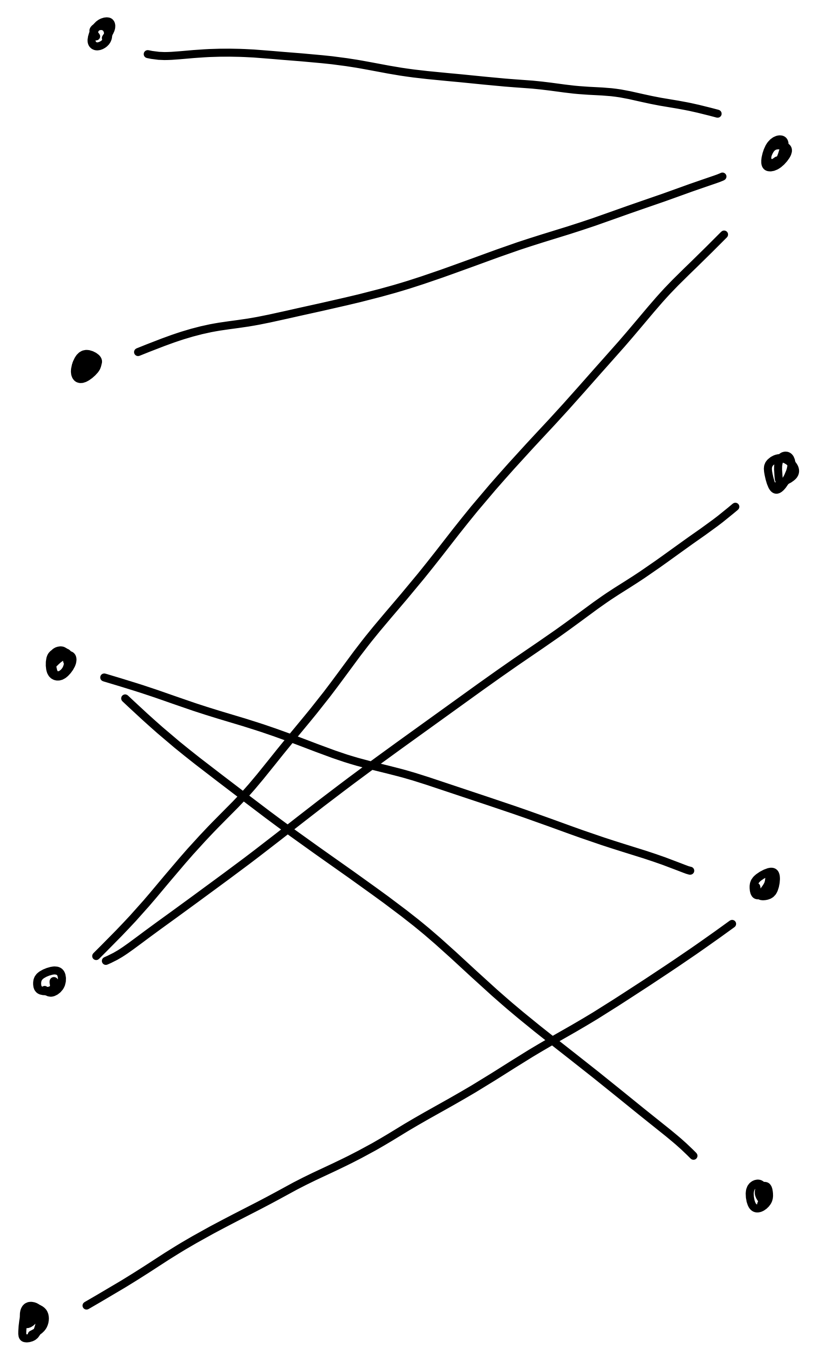
Application: Bipartite matching

- **Input**: A bipartite graph $(V = L \sqcup R, E)$
- **Output**: A maximal collection of edges that don't share any vertices.

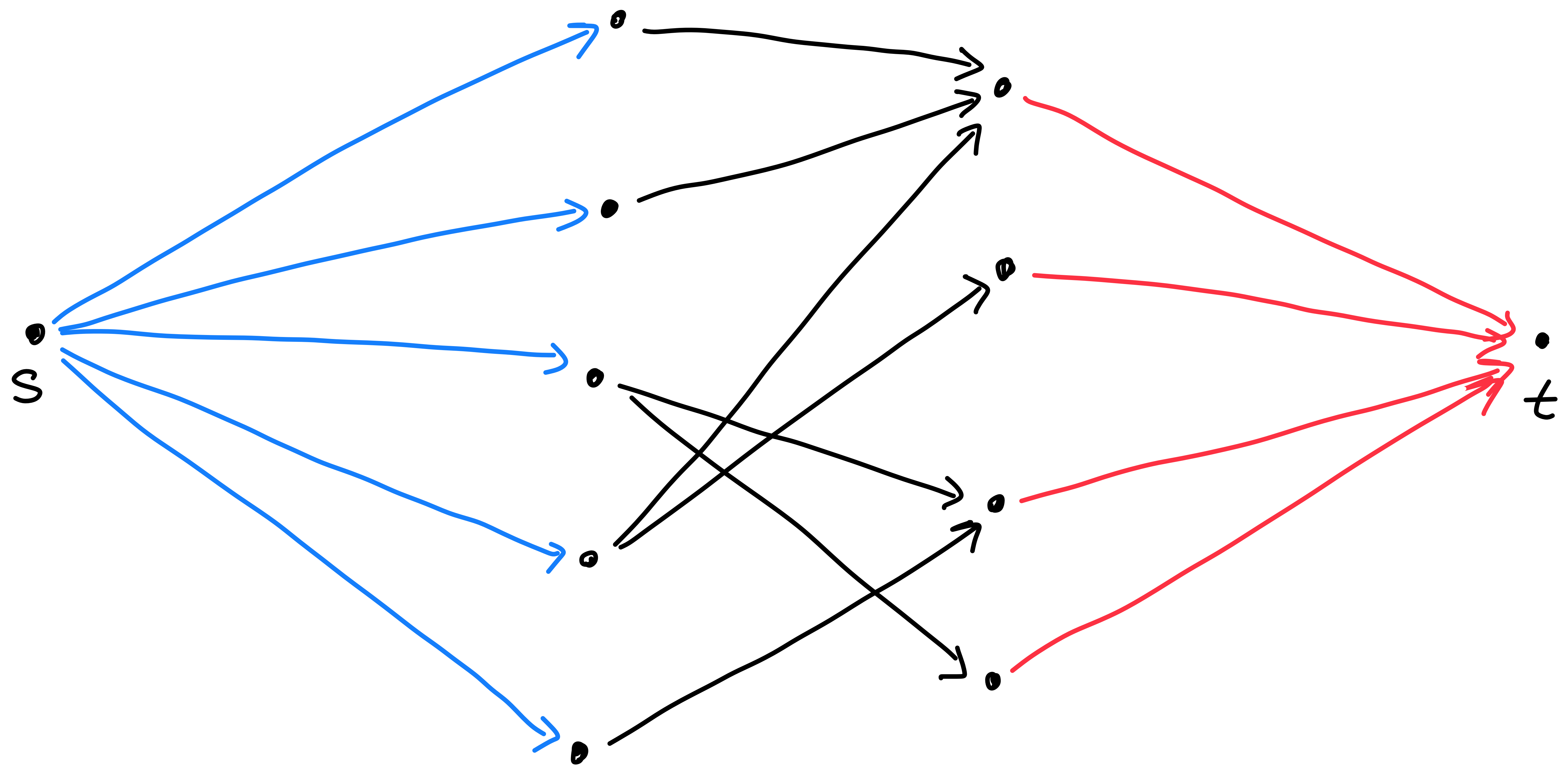
- To solve with Ford-Fulkerson, we need to create a directed graph and identify a source s and sink t.



Application: Bipartite matching



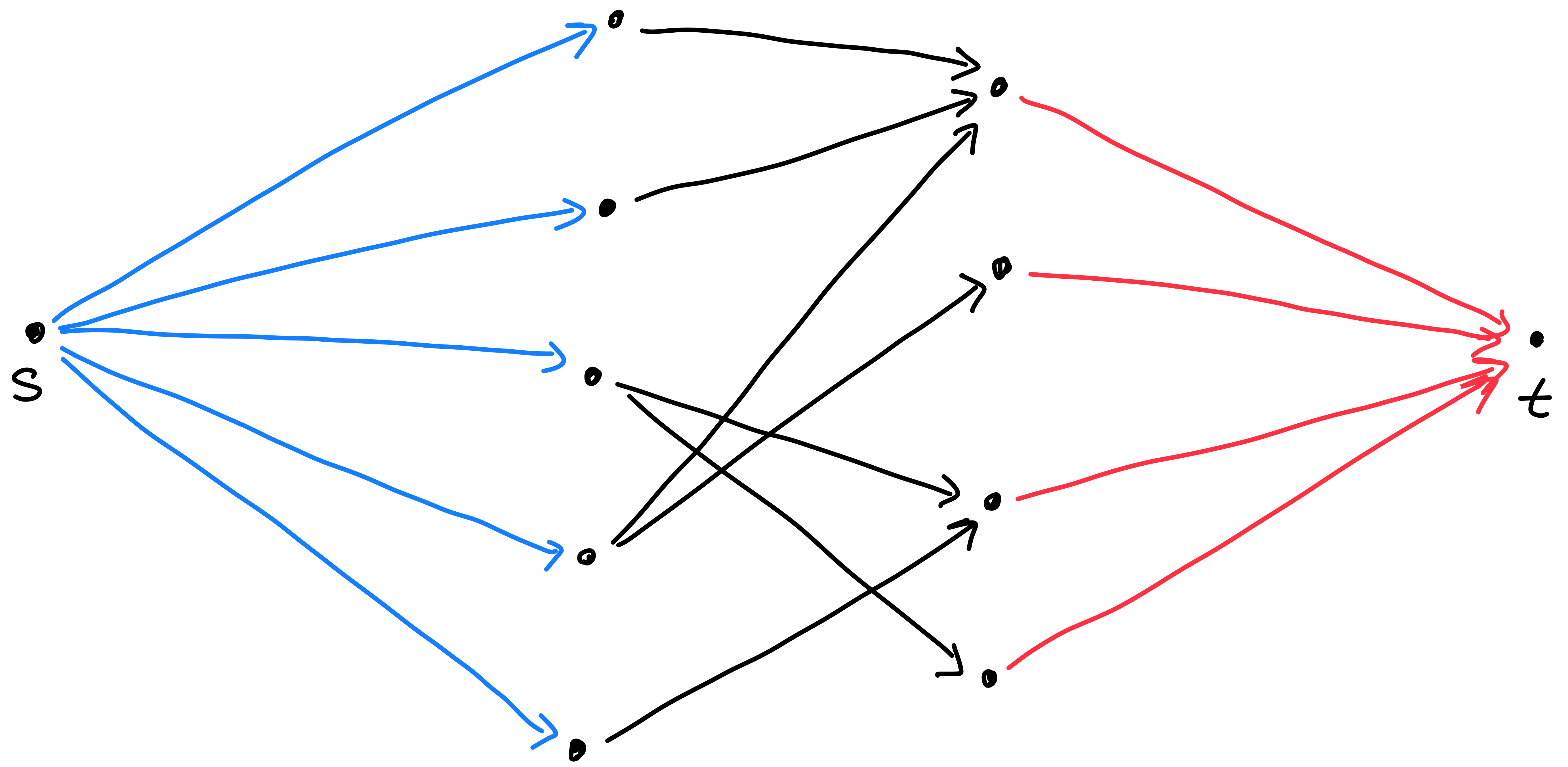
Application: Bipartite matching



all edges of capacity 1

Application: Bipartite matching

Run Ford-Fulkerson on this graph.



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Application: Bipartite matching

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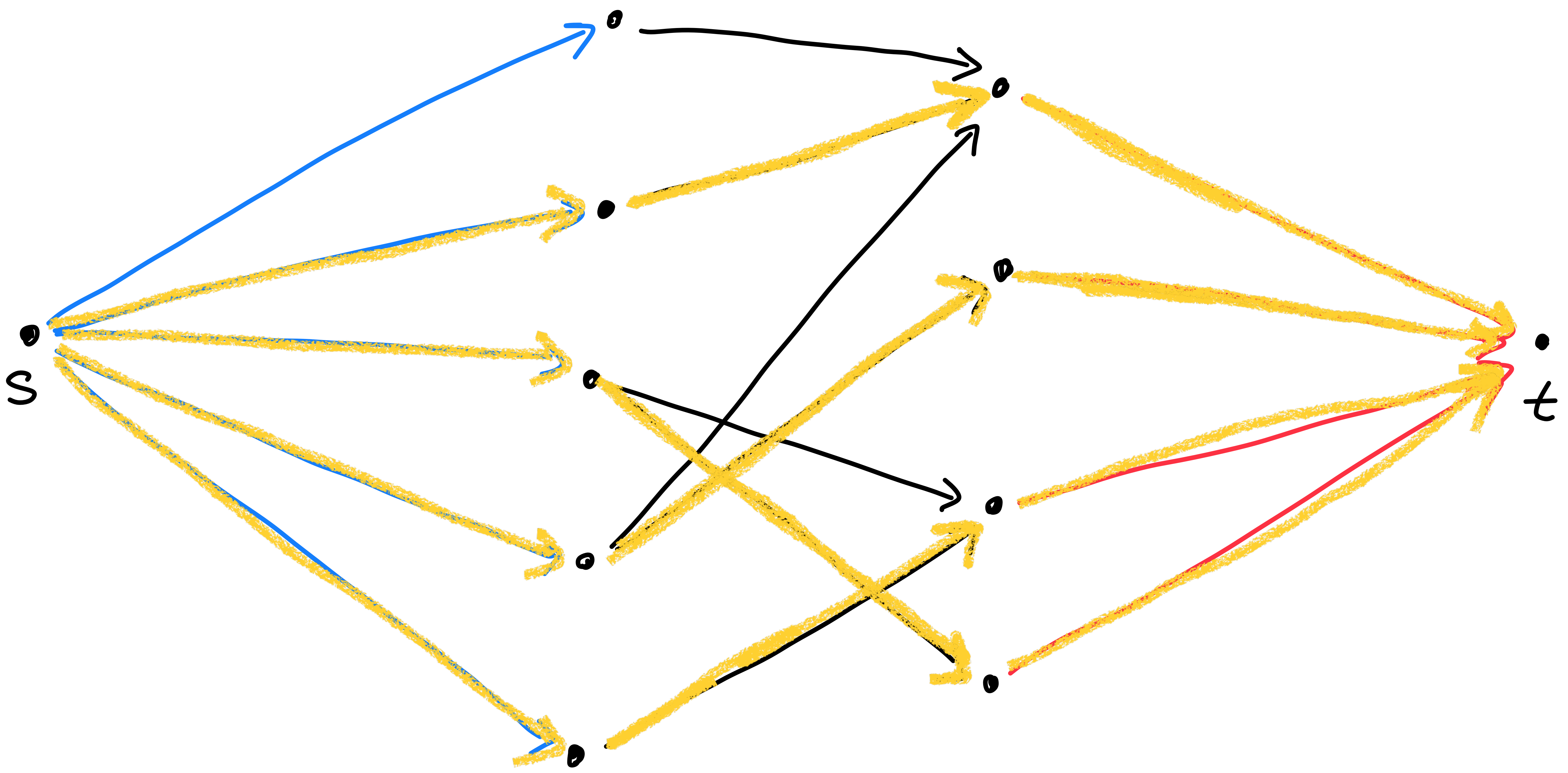


all edges of capacity 1

Application: Bipartite matching

- **Claim:** The edges of flow 1 in the max flow form a maximal bipartite matching.
- **Proof:**
  - Integer flow and bipartite matching *bijection*:
    - Since FF only outputs integer flow, and each edge capacity is 1, at most 1 edge leaving a $v \in L$ can be selected. So a integer flow yields a matching of equal size.
    - For every edge $u \to v$ from L to R in the bipartite matching add the flow $s \to u \to v \to t$. All flows will be compatible. So a bipartite matching yields a flow of equal size.
  - By bijection, max flow will yield a max bipartite matching.

Application: Bipartite matching

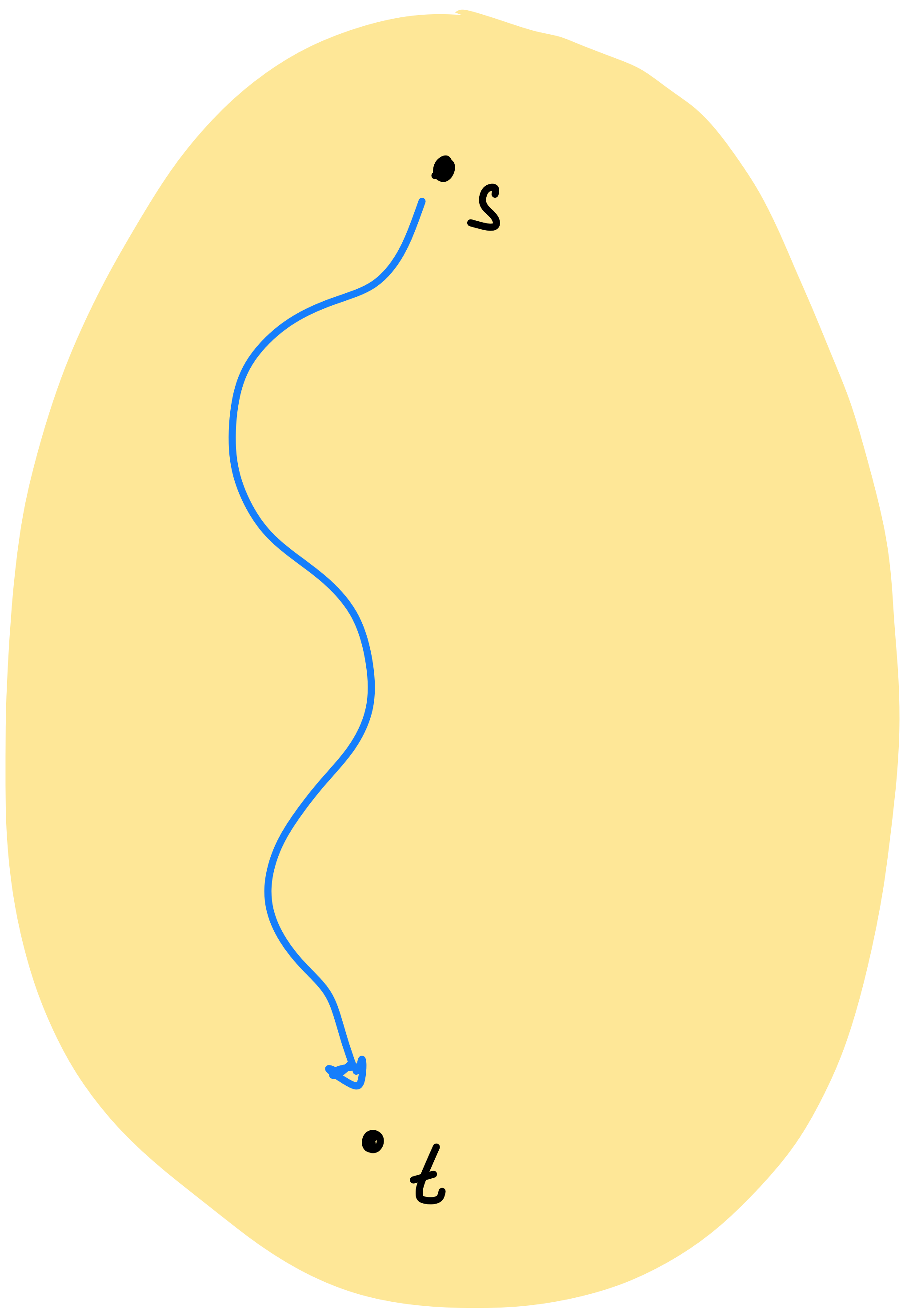
- **Runtime:** Each edge has capacity 1, root node has total output capacity n.
  - $C = n$, number of edges in network is $m + 2n$.
  - Total runtime after reduction, $O((2n + m)n) = O(n^2 + mn)$.

Ford-Fulkerson can be slow

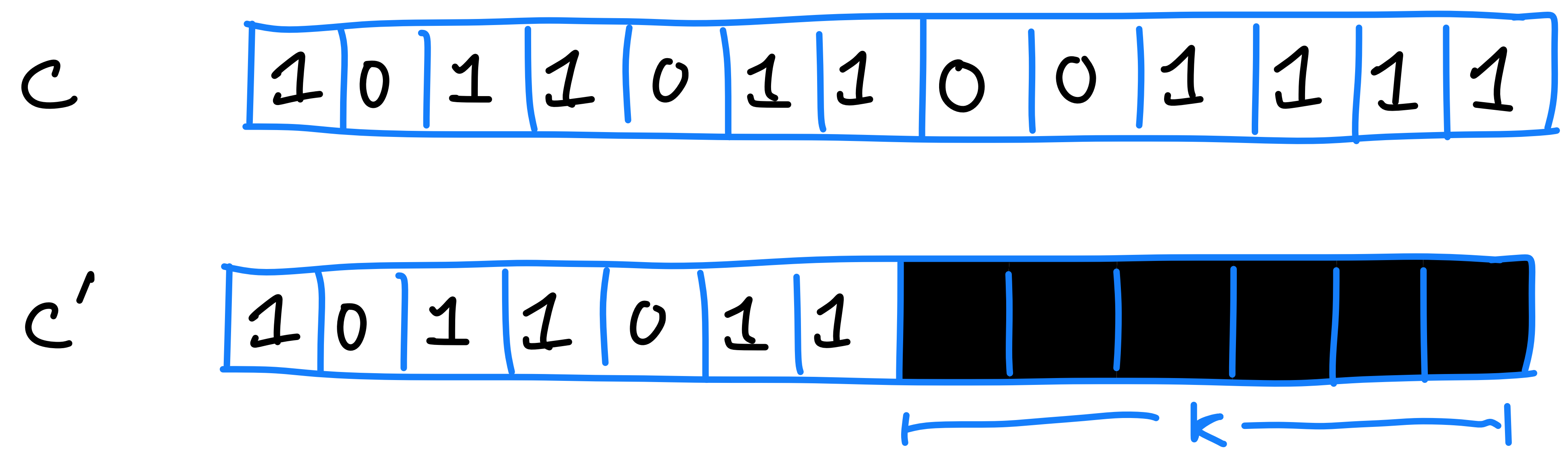
- **Input:** The input is a flow network (G, s, t, c)
  - Formally, $c = \{c(e_1), c(e_2), \ldots, c(e_m)\}$ for each edge e_j with $c(e_j)$ being a number expressed in binary.
  - Then $C = C(\{s\}, V \backslash \{s\})$ is an exponential number in the size of the input.
  - Ford-Fulkerson can be slow! Runtime of $O(mC)$.
    - Because each update only guarantees flow increase by 1.
    - Is there a fast way to find bigger increases in flow?

Finding an augmenting path

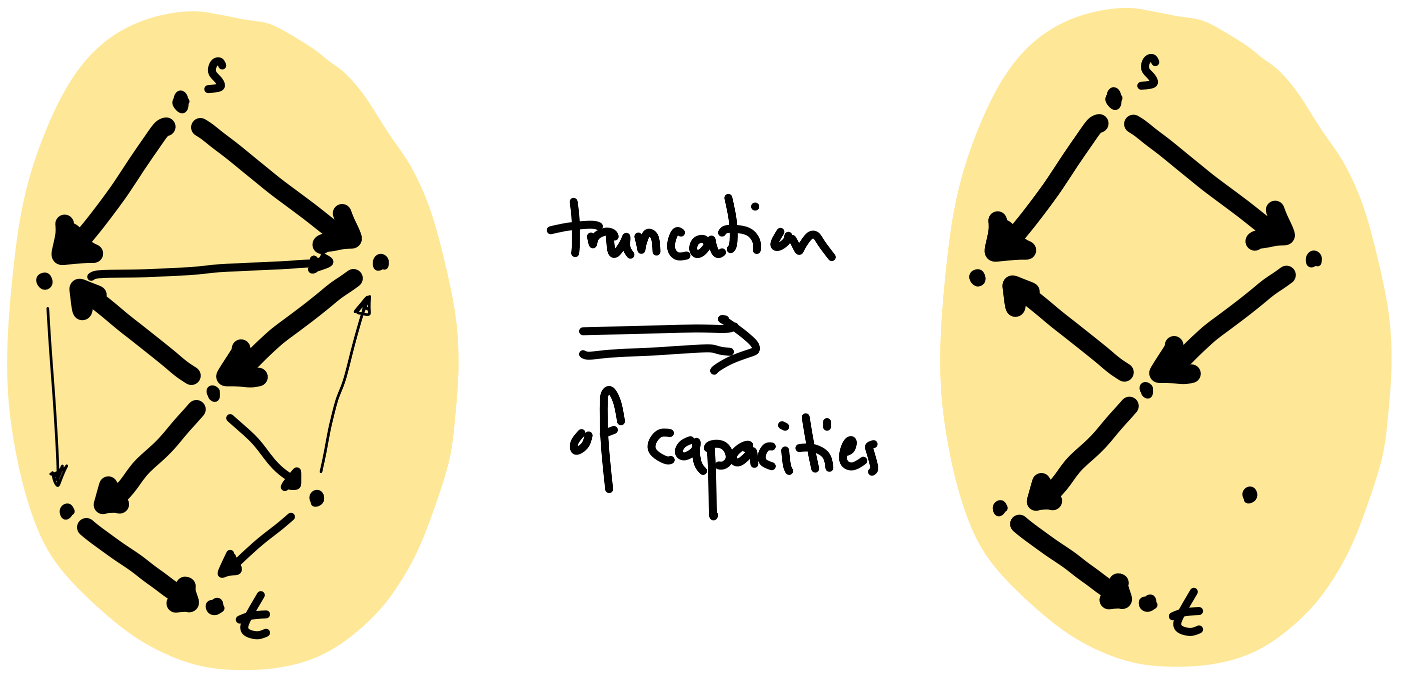
- We previously chose an augmenting path $s \rightsquigarrow t$ in G_f by running a graph traversal from s to t and picking a path
- This will find an augmenting path but may fail to find the augmenting path of largest bottleneck capacity
- **Idea:** If there exists some augmenting path of bottleneck capacity $\geq 2^k$, can we construct an algorithm that finds an augmenting path of bottleneck capacity at least 2^k?



Finding a pretty big augmenting path



- **Fast (Scaling) Augment:** Starting with $k \leftarrow \lfloor \log C \rfloor$,
  - Find an augmenting path of size 2^k:
    - Run regular augmenting path search on G_f except with capacities $c' = \lfloor c/2^k \rfloor$.
    - If a path exists of bottleneck $\geq 2^k$, it still exists in adjusted graph.
  - If yes, add this augmenting path and restart.
  - If not, decrease $k \leftarrow k - 1$, and repeat.



- **Theorem:** If the max bottleneck capacity of any augmenting path is v, the fast augment subroutine finds an augment of size $\geq v/2$ in time $O(m \log C)$.

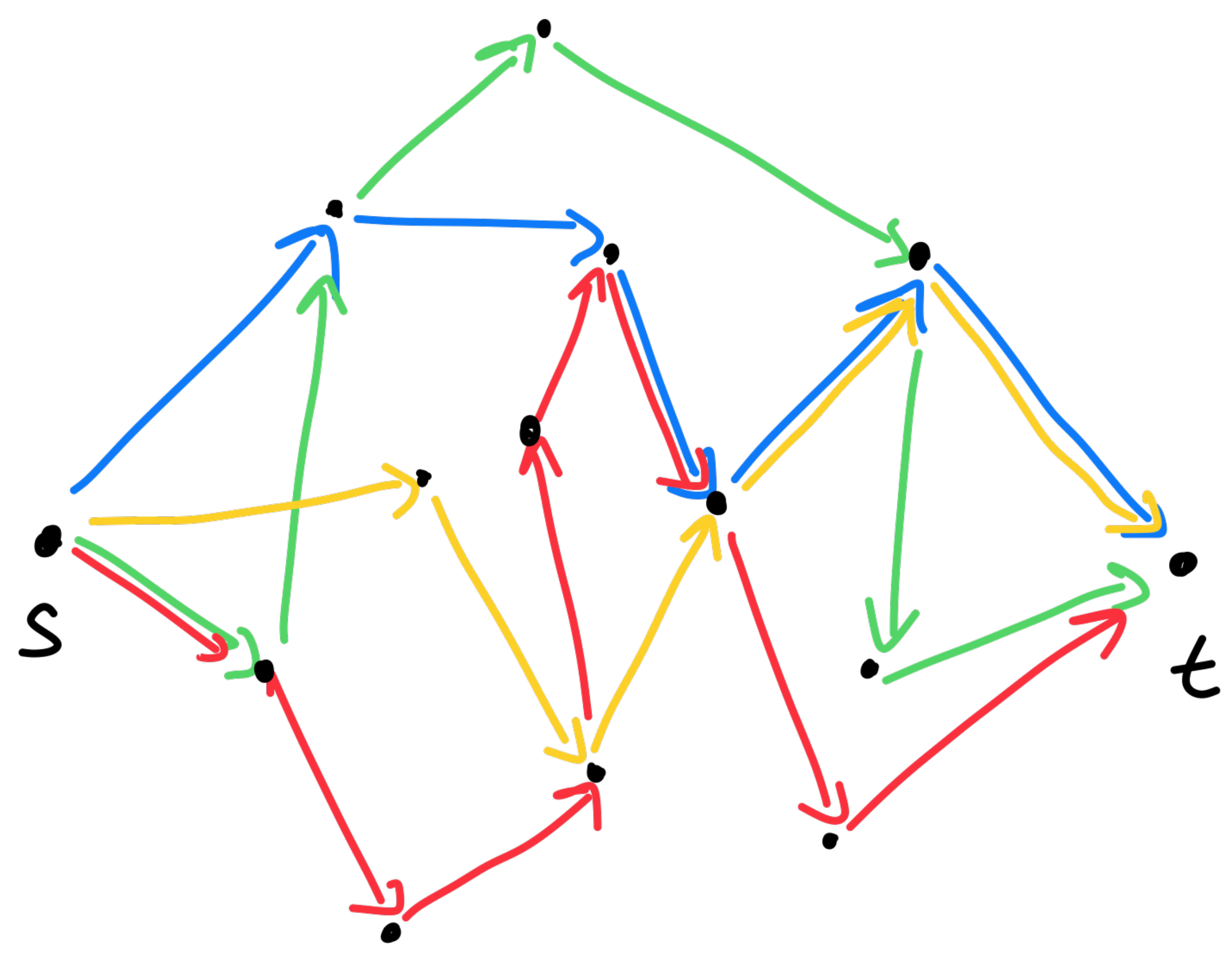
Scaling Ford-Fulkerson

- **Algorithm:** Start with flow $f \leftarrow 0$ and $G_f \leftarrow G$.
  - While the fast augment subroutine can find an augmenting path p
    - Augment f by f_{aug} along path and update G_f
- **Theorem:** The scaling version of Ford-Fulkerson runs in time $O(m^2 \log C)$.

Scaling Ford-Fulkerson runtime

- To prove the runtime of $O(m^2 \log C)$, we need to prove a few lemmas.

- **Lemma:** Every flow f can be expressed as the sum of $\leq m$ flows along paths.
- **Proof:**
  - While there exists a path $p : s \rightsquigarrow t$ in the flow,
    - Remove flow along p of the bottleneck capacity of p.
    - The resulting flow is 0 along some edge.
  - This can be repeated $\leq m$ times.



Scaling Ford-Fulkerson runtime

- To prove the runtime of $O(m^2 \log C)$, we need to prove a few lemmas.

- **Lemma:** Every flow f can be expressed as the sum of $\leq m$ flows along paths.
- **Corollary:** There exists a path within the flow of bottleneck capacity $\geq \text{maxflow}(G)/m$.
- **Proof:**
  - Run the lemma on the max flow.
  - By pigeon-hole principle, one of the paths must have large flow.

Scaling Ford-Fulkerson runtime

- To prove the runtime of $O(m^2 \log C)$, we need to prove a few lemmas.

- **Lemma**: Every flow f can be expressed as the sum of $\leq m$ flows along paths.
- **Corollary**: There exists a path within the flow of bottleneck capacity $\geq \text{maxflow}(G)/m$.
- **Corollary**: Fast-Augment will find an augmenting path in G_f of bottleneck capacity $\geq \text{maxflow}(G_f)/(2m)$.

Scaling Ford-Fulkerson runtime

- **Corollary:** Fast-Augment will find an augmenting path in G_f of bottleneck capacity $\geq \text{maxflow}(G_f)/(2m)$.

- Each iteration of Fast-Augment will decrease by a mult. factor of $1 - 1/(2m)$

- # of iterations $\leq \log_{(1-1/(2m))^{-1}}(C) = \dfrac{\log C}{-\log(1 - 1/(2m))} \leq \dfrac{\log C}{1/(2m)} = 2m \log C$.

- Total runtime is $O(m) \cdot 2m \log C = O(m^2 \log C)$.

Flow independent of capacity

- So far, for integer capacities:
  - **Vanilla Ford-Fulkerson**: Runtime $O(mC)$
    - Pick any augmenting path
  - **Scaling Ford-Fulkerson**: Runtime $O(m^2 \log C)$
    - Pick the largest augmenting paths
  - **Edmonds-Karp (next)**: Runtime $O(m^2 n)$
    - Pick the shortest augmenting path (in terms of # of edges)

Edmonds-Karp algorithm

- Initialize $f \leftarrow 0$ and $G_f \leftarrow G$
- While BFS starting from s outputs a path $p : s \rightsquigarrow t$ in G_f.
  - Compute bottleneck capacity b and update f and G_f by augmenting f along p at capacity b.
- Output resulting flow f.

Edmonds-Karp

- We know the algorithm: it's BFS based-augumentations.
    - Each run of BFS will compute an augmentation in time $O(m)$.
    - I've claimed the runtime is $O(m^2n)$.
    - Therefore, we need to be able to prove that only $O(mn)$ augmentations are needed.

Edmonds-Karp

- Every time an augmenting path is chosen, the bottleneck edge e becomes saturated — i.e. $f(e) = c(e)$
- Let's show that each edge e can only be the bottleneck in at most $n/2$ augmenting paths.
- Since there are m edges, this yields a max of $\frac{mn}{2}$ augmenting paths.
- Details will be a problem set problem!

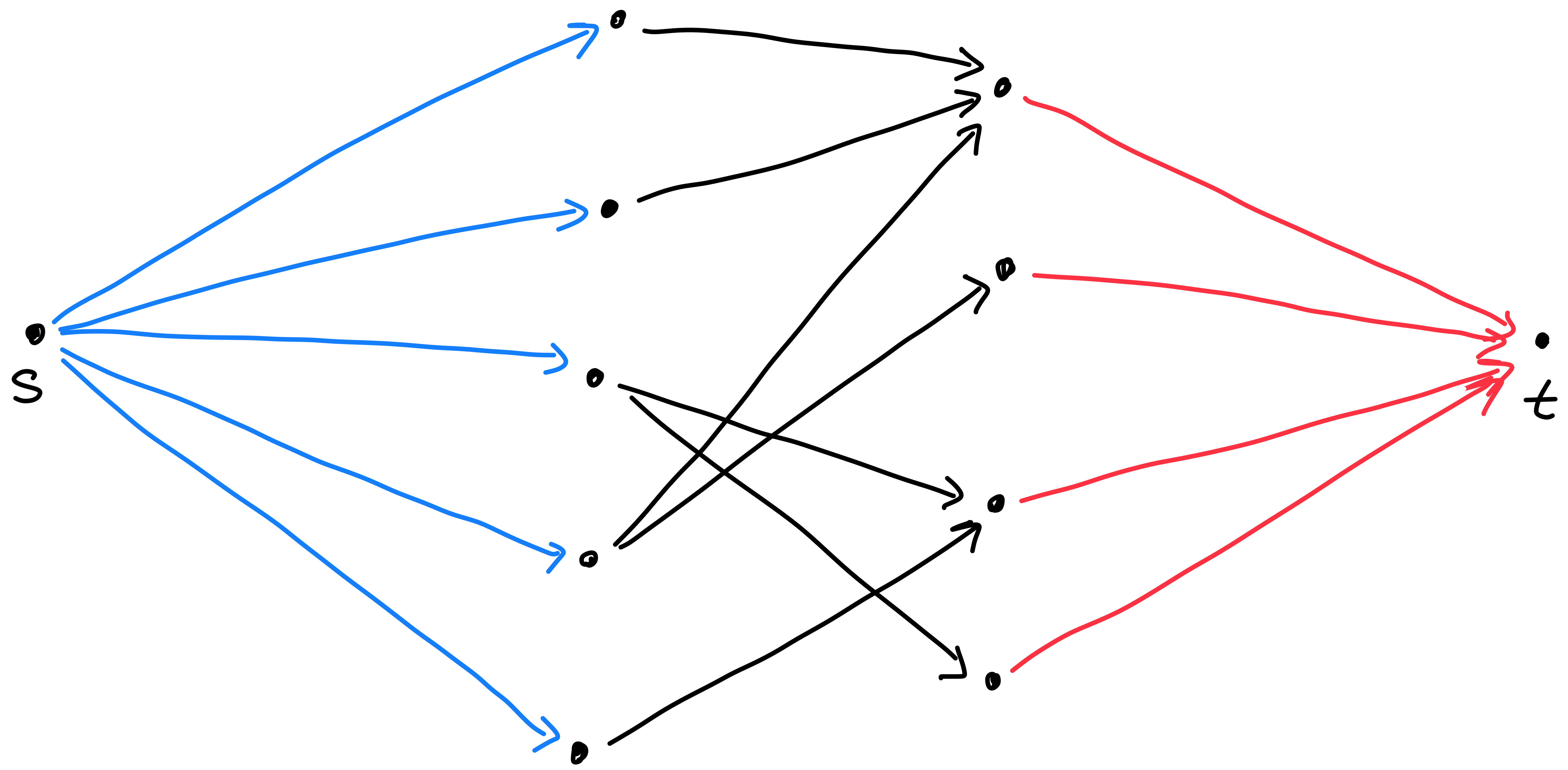
Maximum flow algs are minimum cut algs

- Given a maximum flow f in a network G, the set of edges that are saturated: $f(e) = c(e)$ form a minimum cut
    - The min cut may not be unique just as the max flow may not be unique
- Maximum flow and minimum cut are dual problems
    - Two sides of the same coin
    - We will see this come up again in a couple of weeks!

Applications of max flow/min cut

Recall: bipartite matching

Run Ford-Fulkerson on this graph.



all edges of capacity 1

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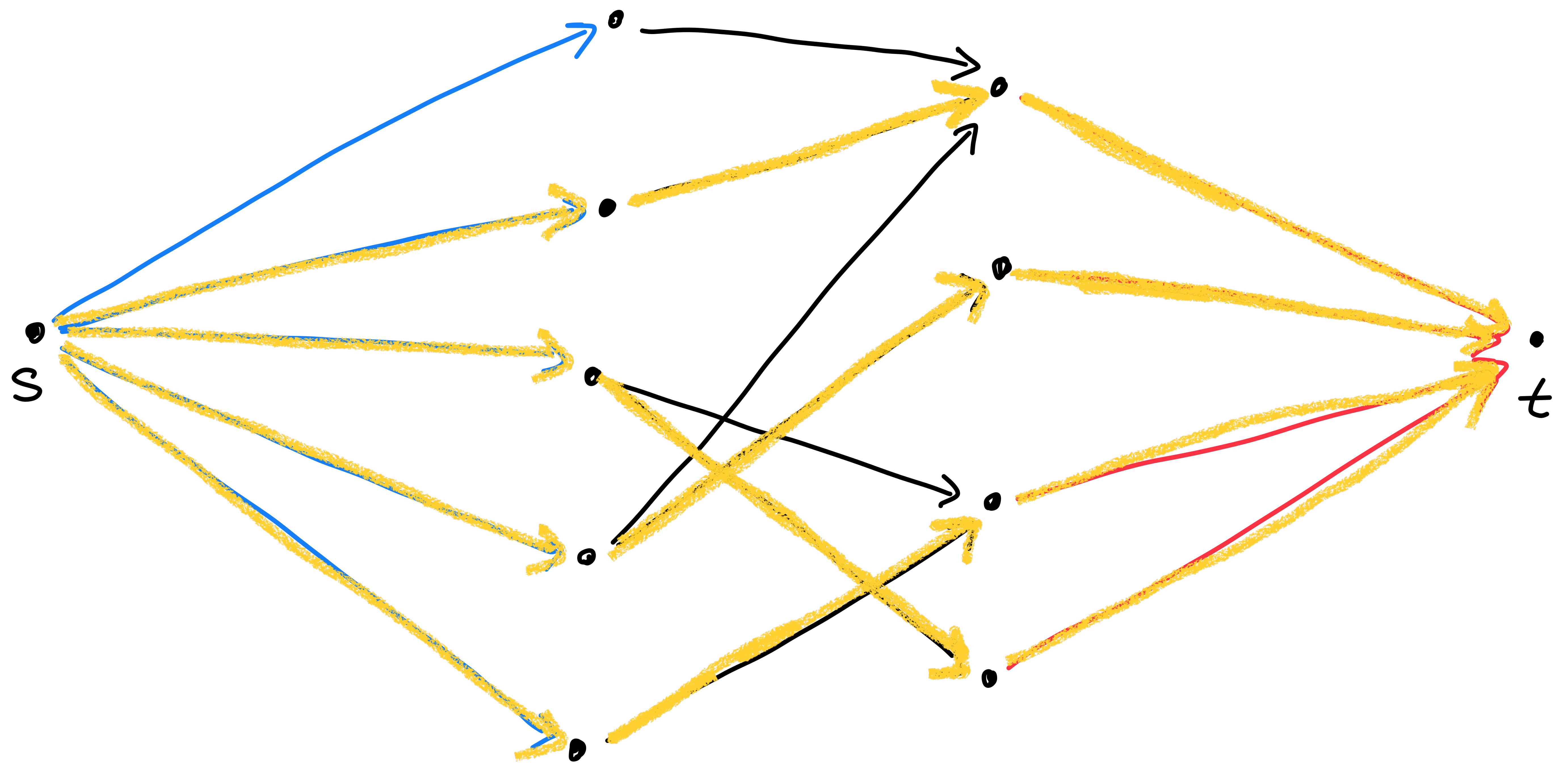

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Recall: Bipartite matching

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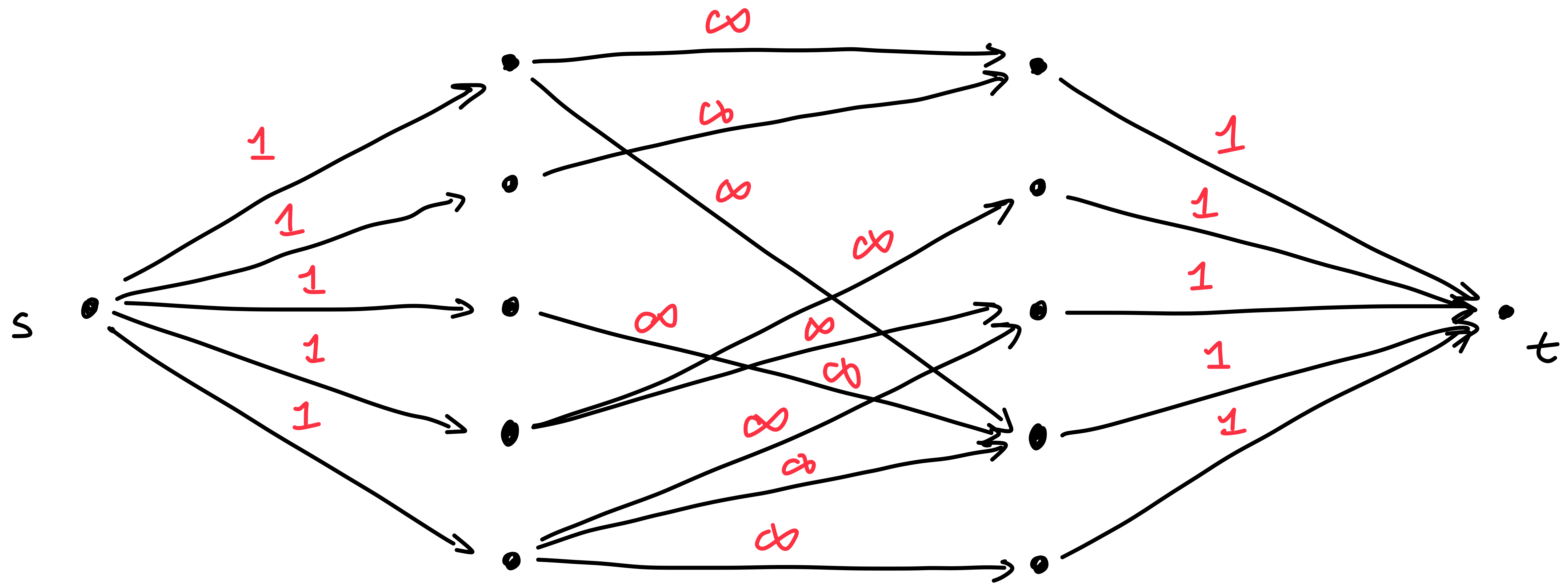
    - For every edge $u \to v$ from L to R in the bipartite matching add the flow $s \to u \to v \to t$. All flows will be compatible. So a bipartite matching yields a flow of equal size.
  - By equivalence, max flow will yield a max bipartite matching.

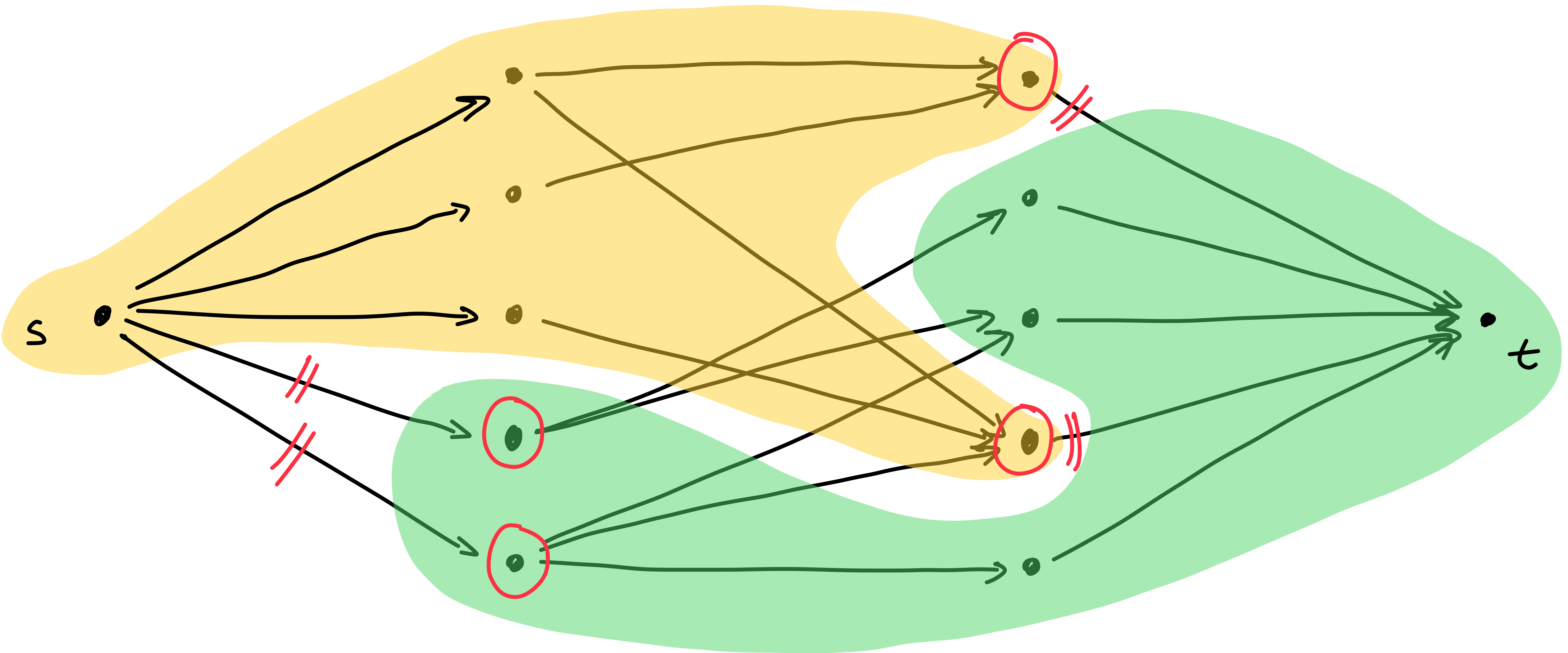
Min cut perspective

- We could solve the same flow problem if we set the capacity to the edges out of s and into t as 1 and set the middle edges to capacity ∞.



Min cut perspective

- Vertices of G involved in the min cut (one per edge crossing the cut) forms a minimum size set of vertices of G that block all flow from s to t



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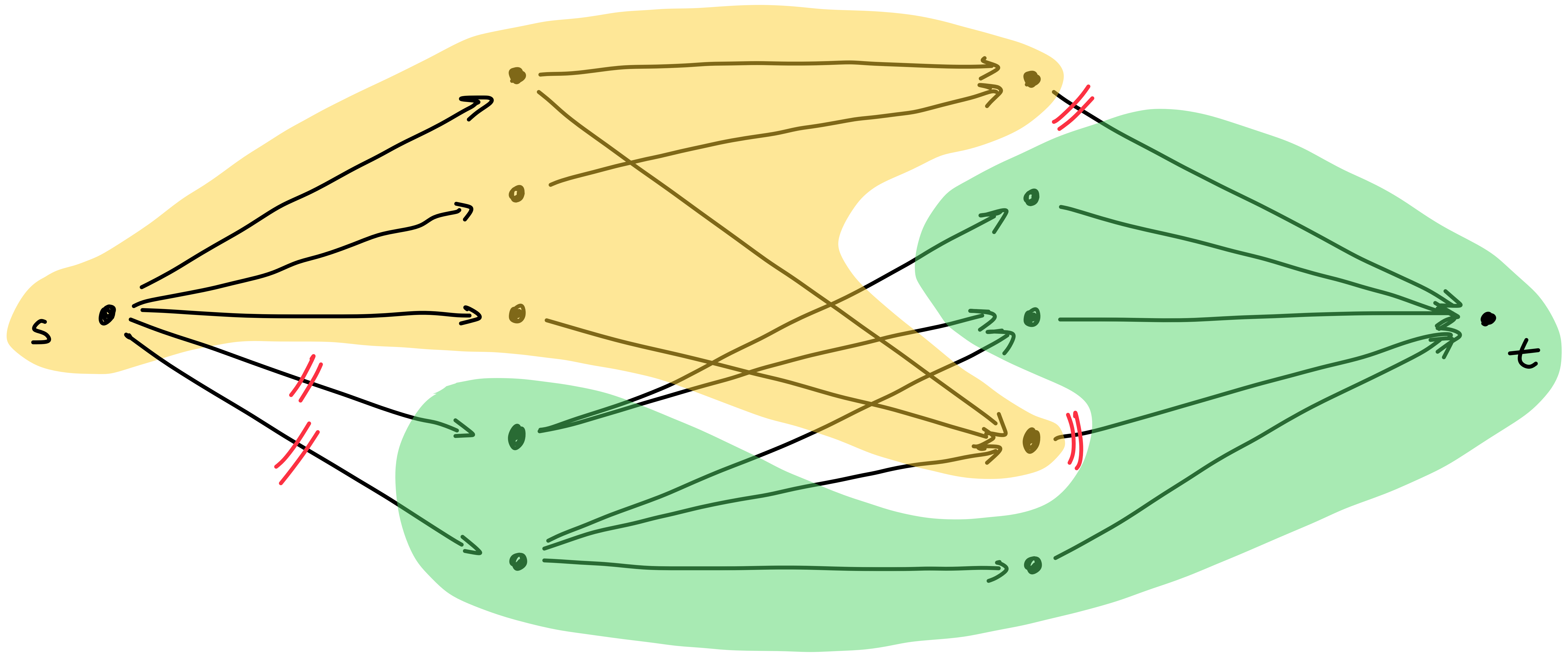
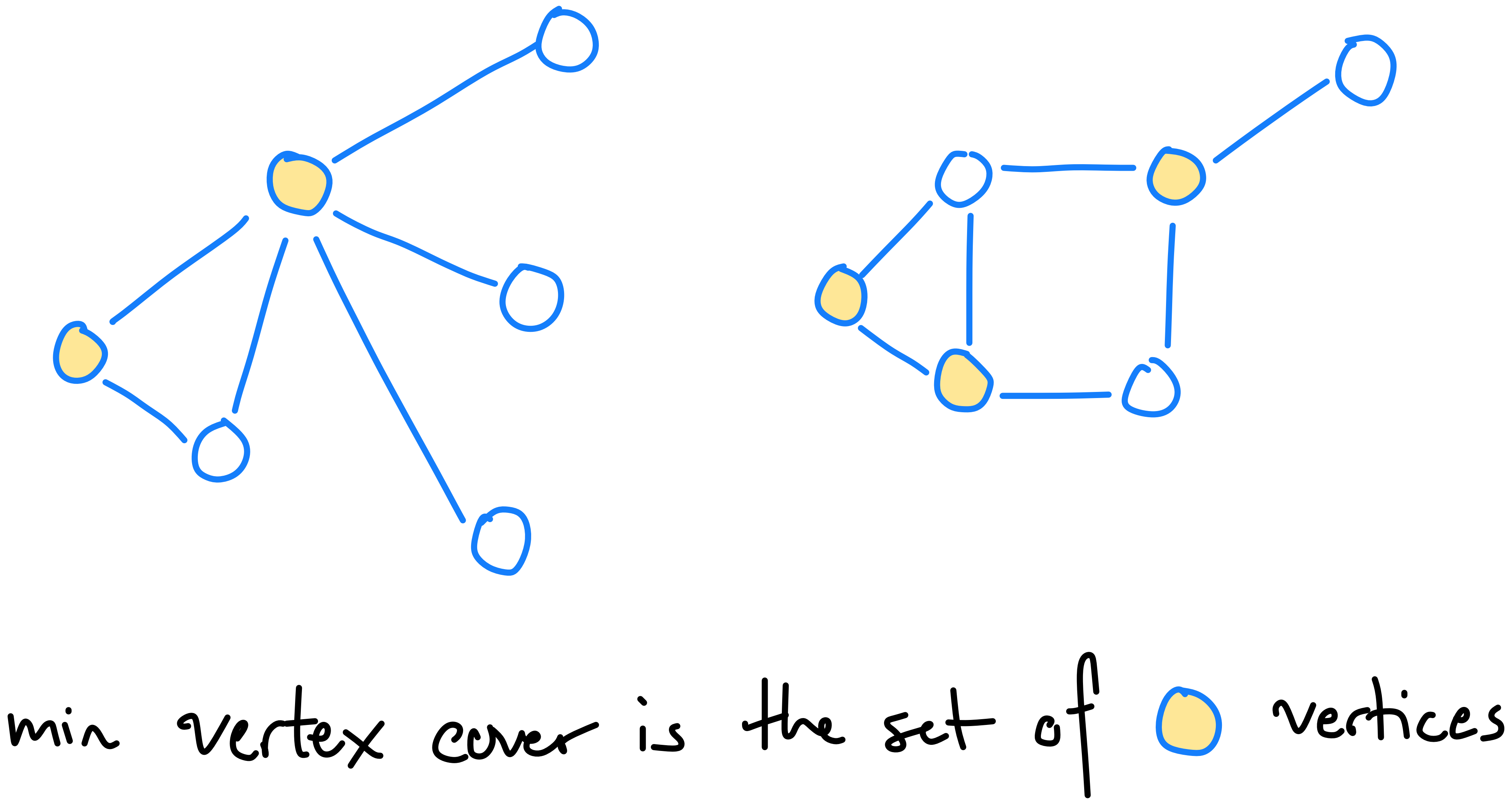


since middle edges have capacity ∞, no middle edges cross the cut.

Minimum vertex cover problem

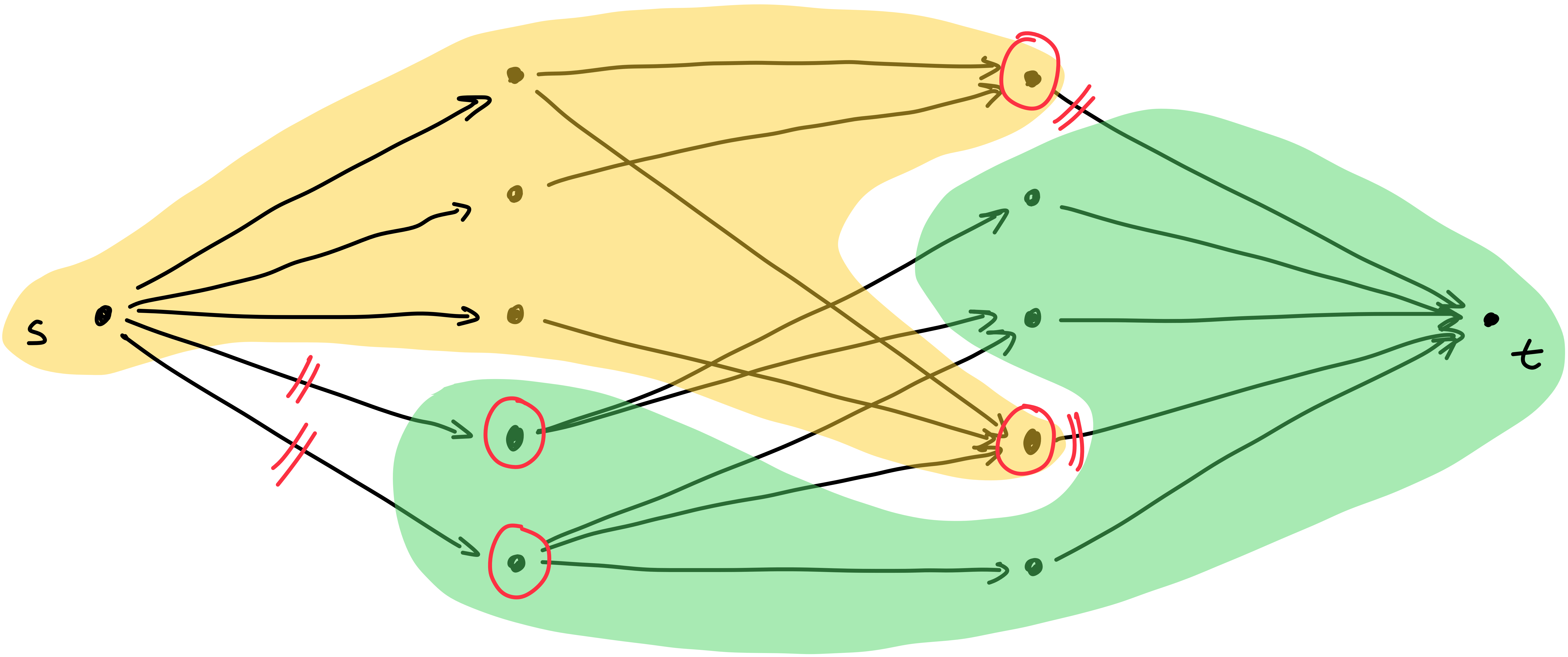
- **Definition:** A subset of vertices $C \subseteq V$ is a *vertex cover* of an undirected graph $G = (V, E)$ iff every edge is touched by some vertex in C.
  - V is a trivial vertex cover for G.
- **Input:** An undirected graph $G = (V, E)$
- **Output:** A minimal vertex cover C for G.



- Min Vertex Cover is a NP-complete problem
- However, min vertex cover on bipartite graphs is efficient!

Minimum vertex cover problem
Bipartite graphs

- **Claim:** The min cut we observed just a minute ago generates a vertex cover.



Minimum vertex cover problem

Bipartite graphs

- **Claim:** The min cut we observed just a minute ago generates a min vertex cover.
- **Proof:**
- Suppose it did not generate a vertex cover.
  - Then there is an edge $e = (u, v)$ not covered. We can augment the flow along the path $s \to u \to v \to t$, a contradiction.
- Suppose there is a smaller min vertex cover C'
  - Then the edges connecting s or t to C' form the crossing edges of a smaller min cut. A contradiction.

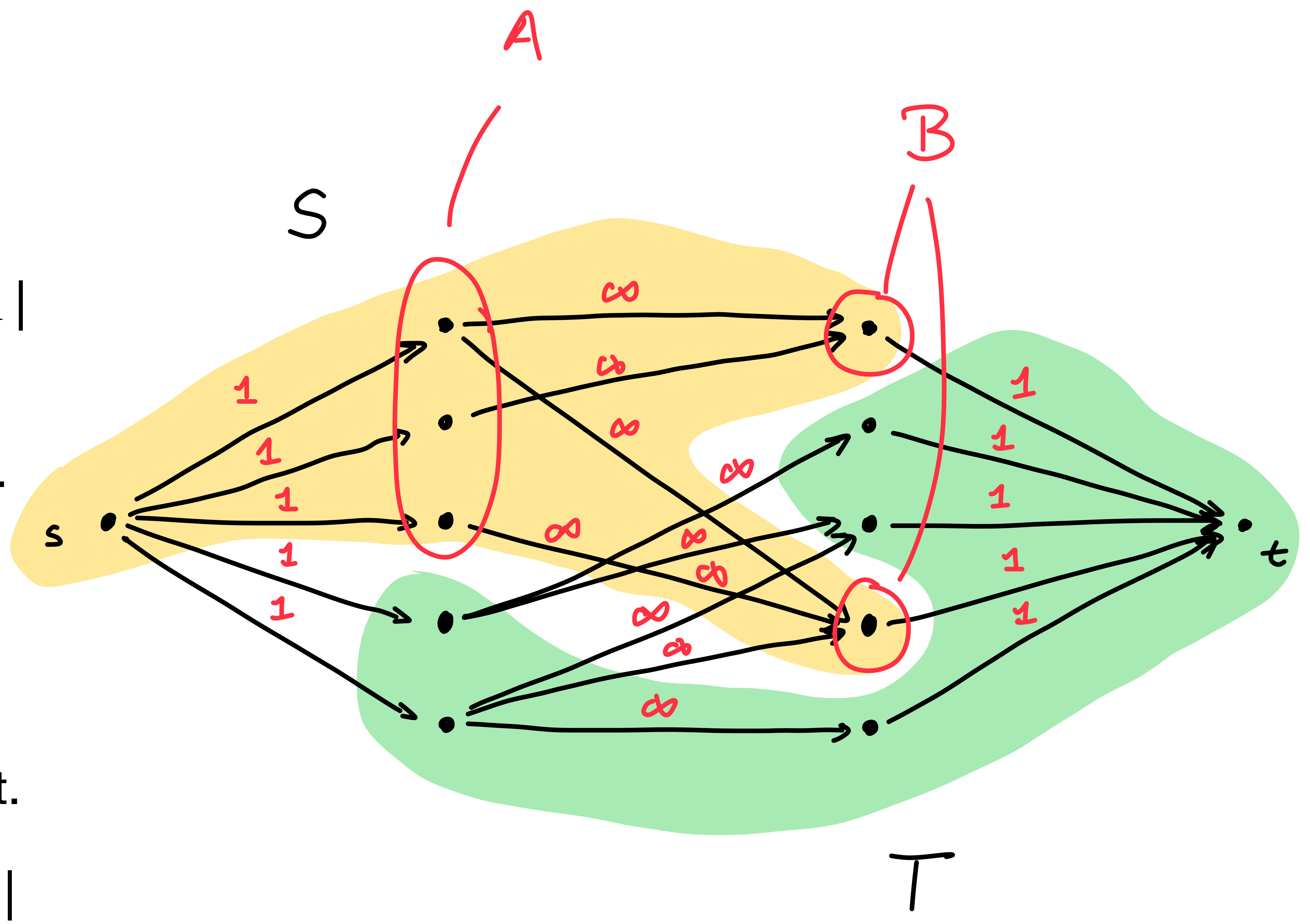
Perfect Matching

- **Definition:** A matching $M \subseteq E$ is perfect iff every vertex participates in some edge of M.
- The previous algorithms give us an algorithm for computing a maximal matching for a bipartite graph.
  - The matching is *perfect* if the size of the matching equals $|L| = |R|$.
  - However, it also provides a criterion for whether a bipartite graph has a perfect matching: **Hall's theorem**.

Hall's theorem

neighbors of the set A in the graph

- **Theorem**: If $|N(A)| \geq |A|$ for all subsets $A \subseteq V$, then there is a perfect matching.
- **Contrapositive**: If there is no perfect matching, then $|N(A)| < |A|$ for some subset A.
- **Proof**: No perfect matching $\implies$ min cut is $< |L|$ in flow network.
  - Let (S, T) be a s-t cut with $c(S, T) < |L|$
  - Choose $A = S \cap L, B = S \cap R$.
  - Then $N(A) \subseteq B$ since no edges across the middle are in the cut.
  - So $|L| > c(S, T) = |L| - |A| + |B| \geq |L| - |A| + |N(A)|$
  - So $|N(A)| < |A|$.

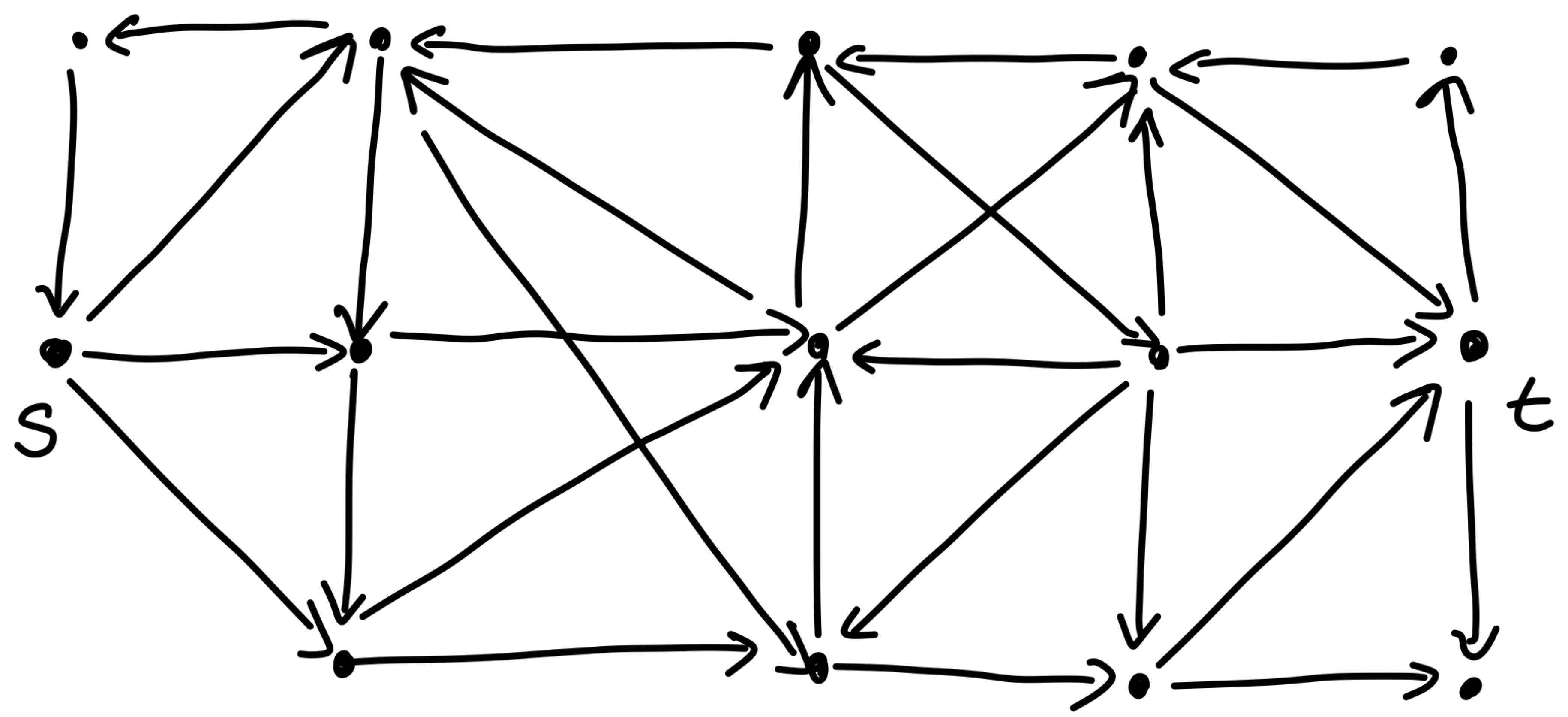


Maximum matching in general graphs

- Bipartite maximum matching runtimes:
  - Generic augmenting path: $O(mn)$
  - State of the art algorithm run in time $O(m^{1+o(1)})$ time with high probability
- General matching algorithm:
  - Solved — $O(mn^{1/2})$ time algorithm exists by Micali-Vazirani
  - Beyond the scope of this course

Edge disjoint paths

- **Input:** A directed graph $G = (V, E)$ with identified vertices s, t
- **Output:** A *maximal* collection of paths $s \rightsquigarrow t$ that share no edges
- **Application**: routing transmissions in communication networks



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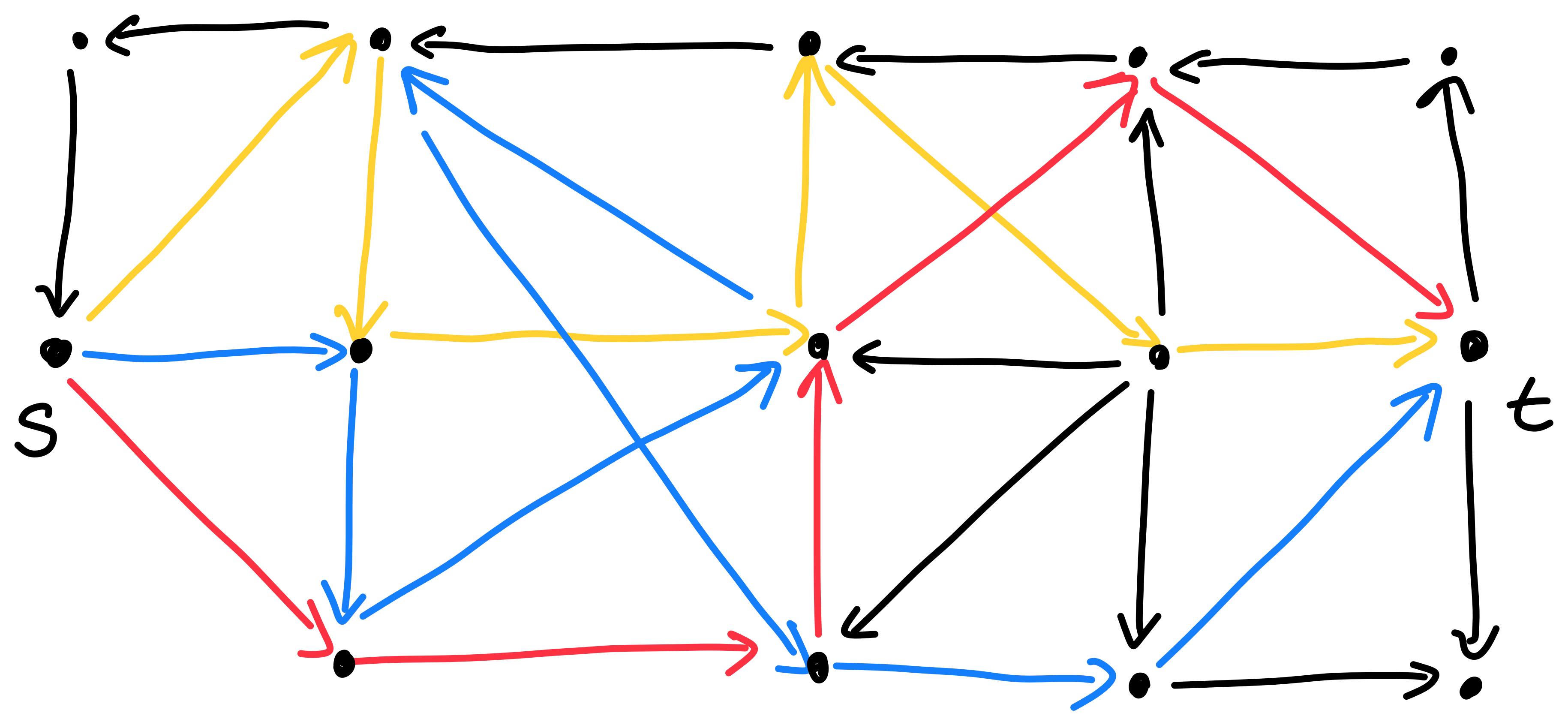
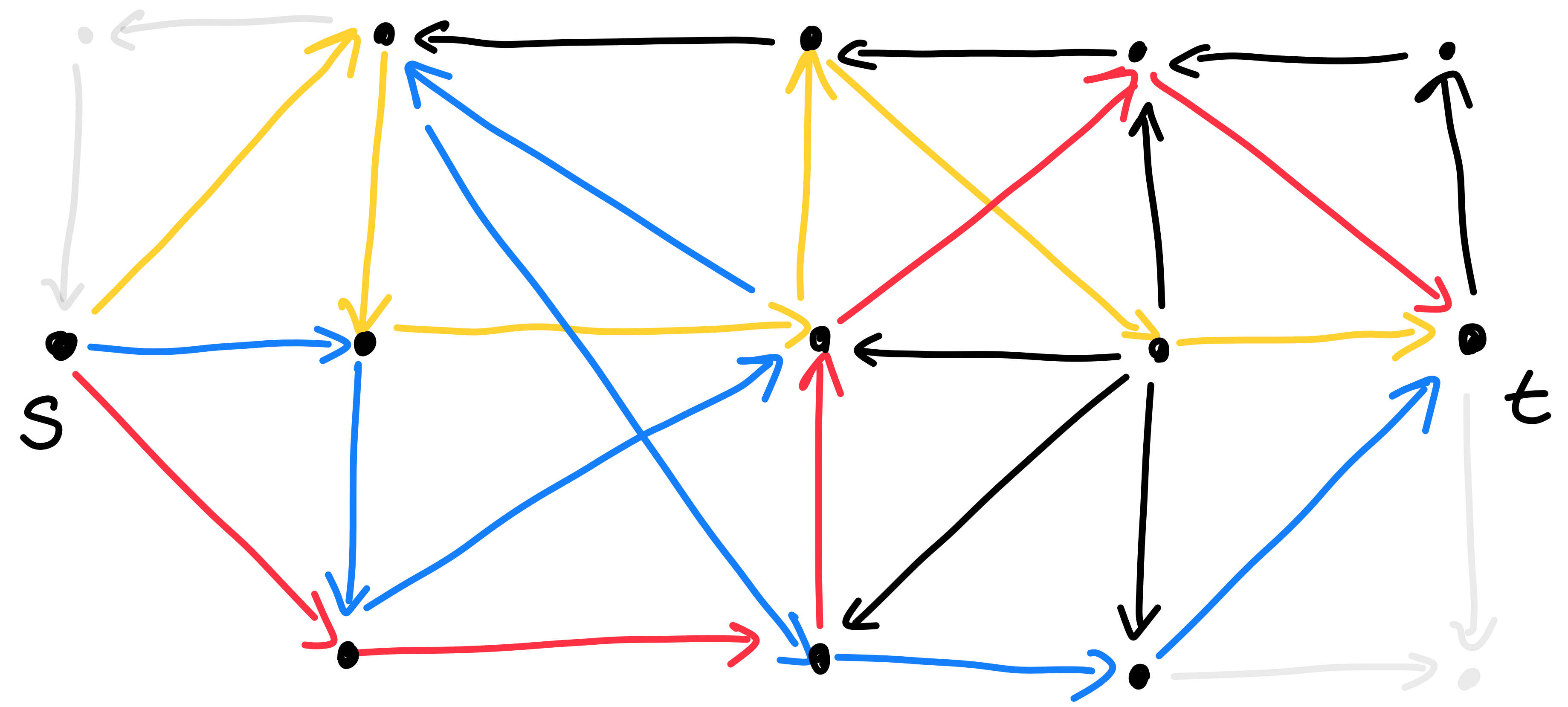
- **Output**: A *maximal* collection of paths $s \rightsquigarrow t$ that share no edges
- **Application**: routing transmissions in communication networks



Edge disjoint paths

- **Idea:** Use max flow to calculate edge disjoint paths
- Need to convert our graph to a flow network
  - Remove any edge $\cdot \to s$ and $t \to \cdot$
  - Set capacity of all remaining edges to 1



- **Correctness argument**: Prove a *bijection* between integer flows and edge disjoint paths. Then maximality of flow yields maximal edge disjoint paths.

Edge disjoint paths

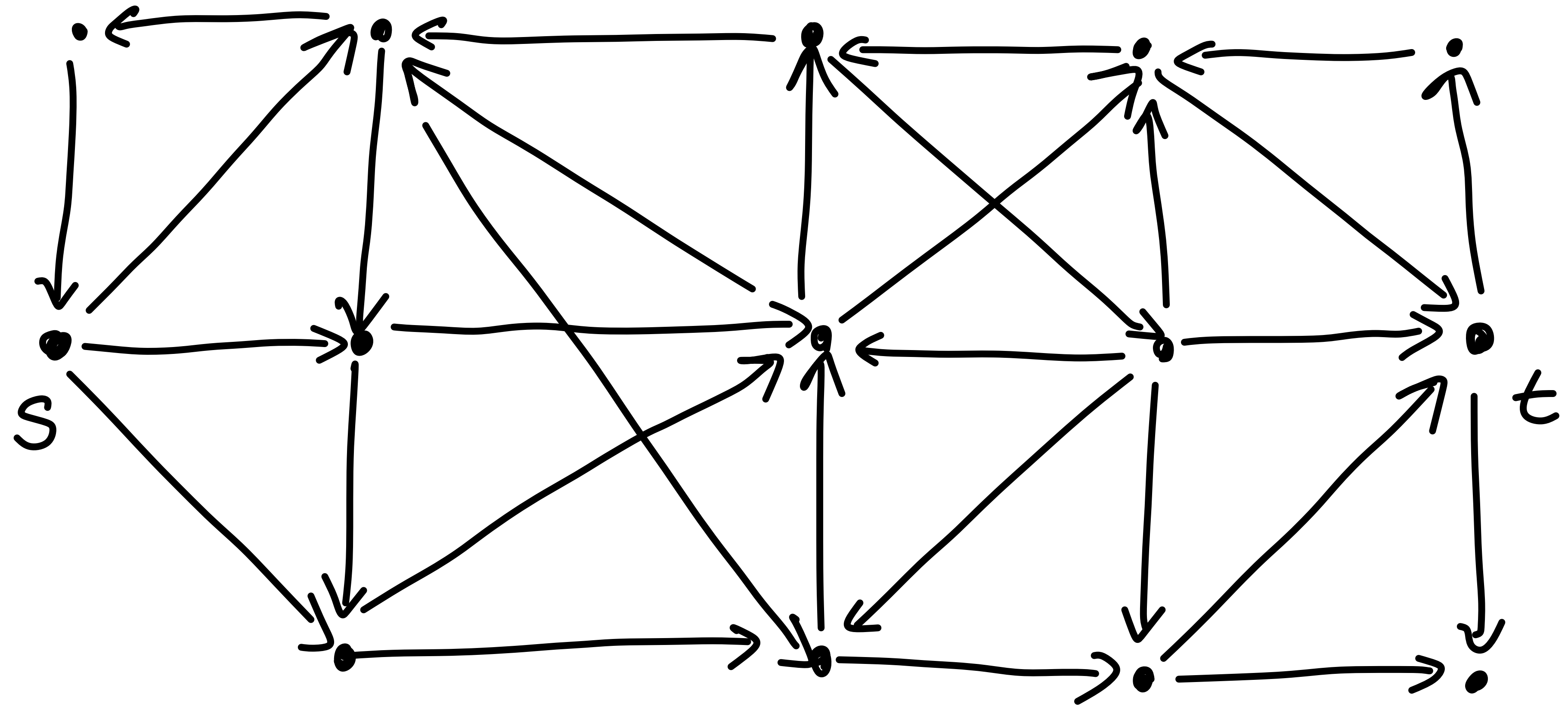
- **Lemma**: Every integer flow is the sum of 1-flow along edge disjoint paths.
- **Proof**:
  - Since capacities are 1, $f(e) \in \{0,1\}$ since it is integer.
  - Then for each edge e, at most one flow along a path can use e.
  - We previously proved that every flow can be decomposed into $\leq m$ paths.
  - Therefore, the paths founds are edge disjoint.

Edge disjoint paths

- **Theorem:** There is a bijection between integer flows and edge disjoint paths.
- **Proof:**
  - Previous lemma converts each integer flow into an edge disjoint path.
  - Sending 1-flow along each edge disjoint path is a valid flow.
    - Conservation of flow follows at every vertex $v \in V \setminus \{s, t\}$ from that of paths.
    - Capacity constraints follow from being a 1-flow and edge disjoint.
  - Together, this proves both directions of the bijection.

Network connectivity

- **Definition**: A set of edges $F \subseteq E$ **disconnects** the source and sink if every path $s \rightsquigarrow t$ must use one edge from F.
- **Input**: directed graph $G = (V, E)$ with source s and sink t
- **Output**: a *minimal* set of edges F that disconnect the source and sink



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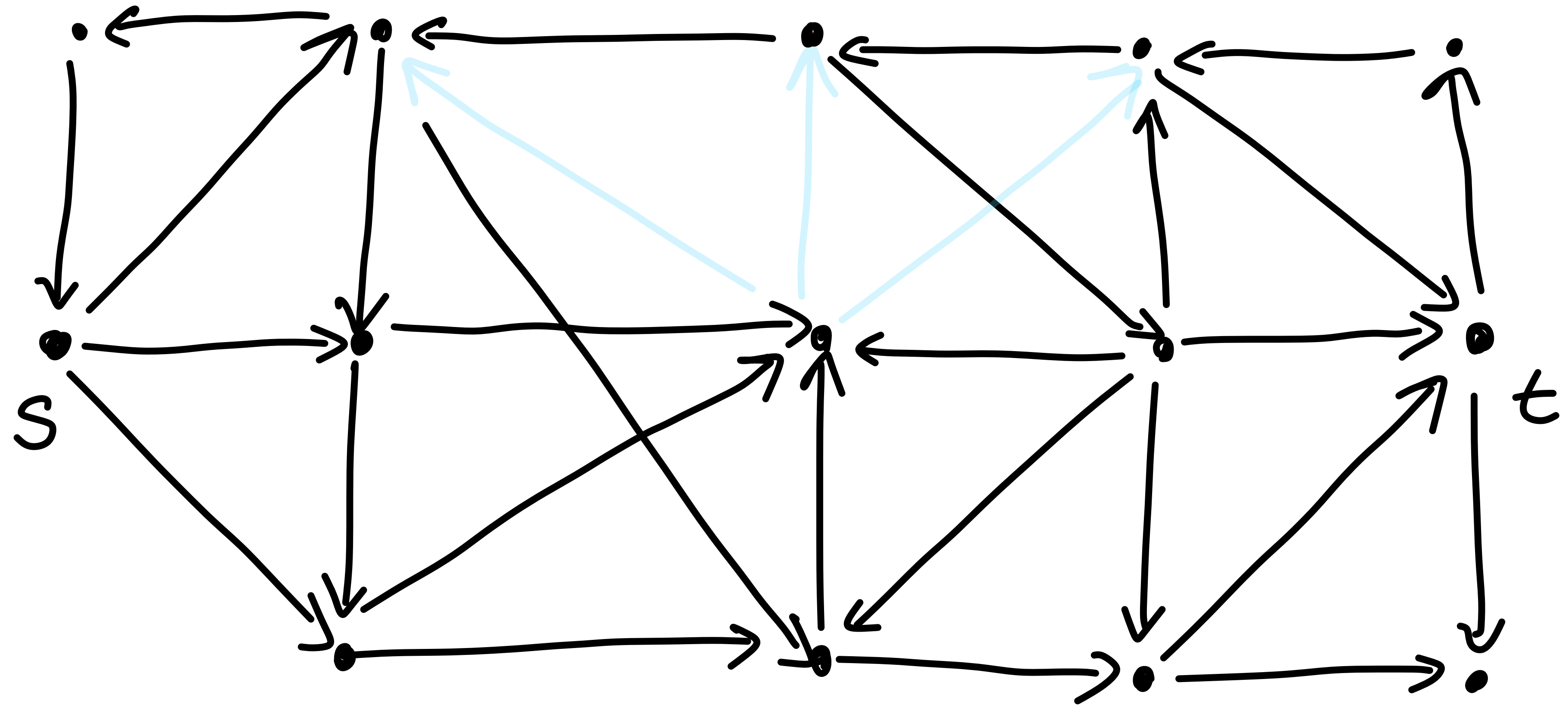
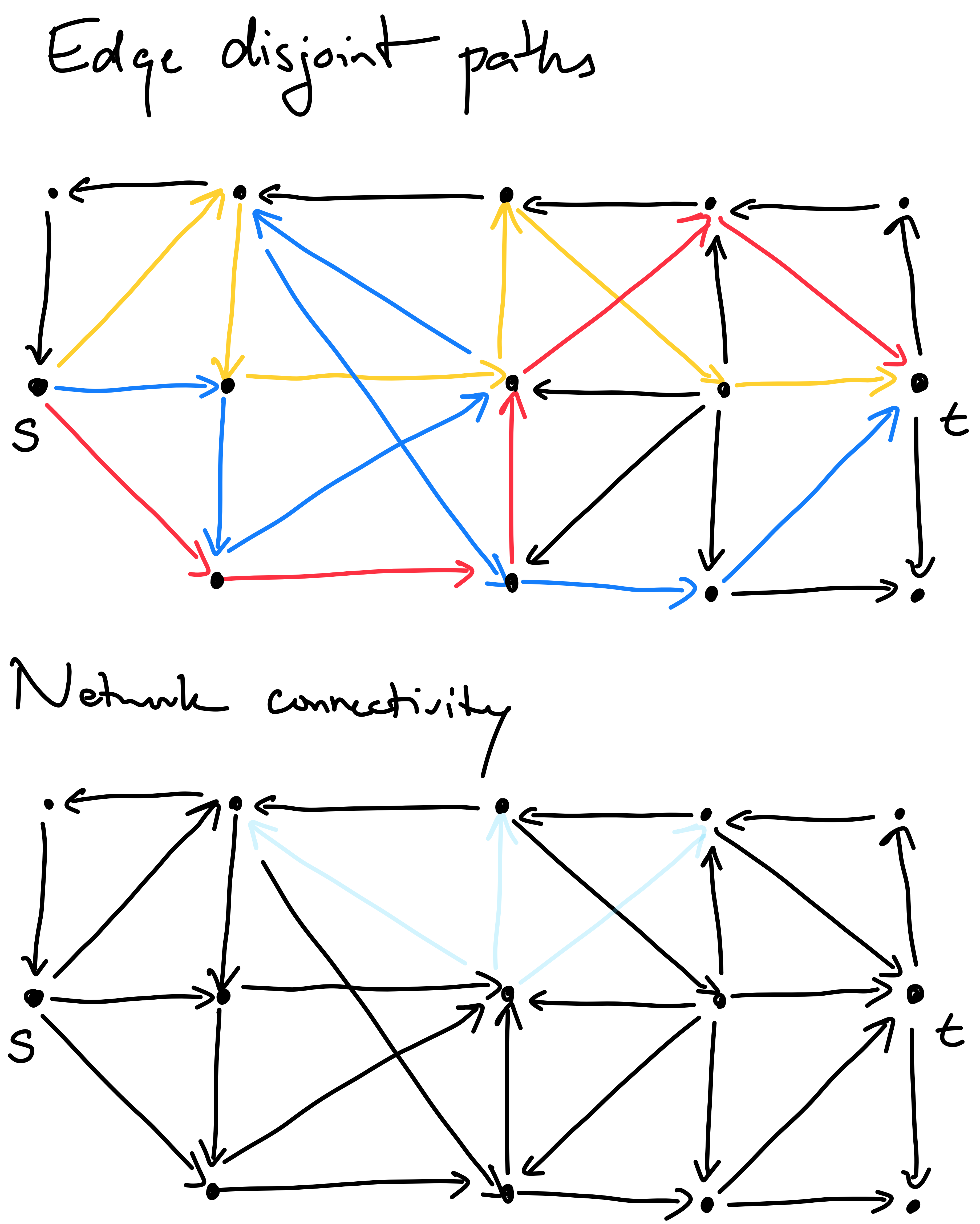


Network connectivity

- **Idea:** Use min cut to calculate minimal network disconnecting set
- Again, need to convert our graph to a flow network
  - Remove any edge $\cdot \to s$ and $t \to \cdot$
  - Set capacity of all remaining edges to 1
- **Correctness argument**: Prove a *bijection* between cuts and network disconnecting sets. Then minimality of cut yields minimal disconnecting set.

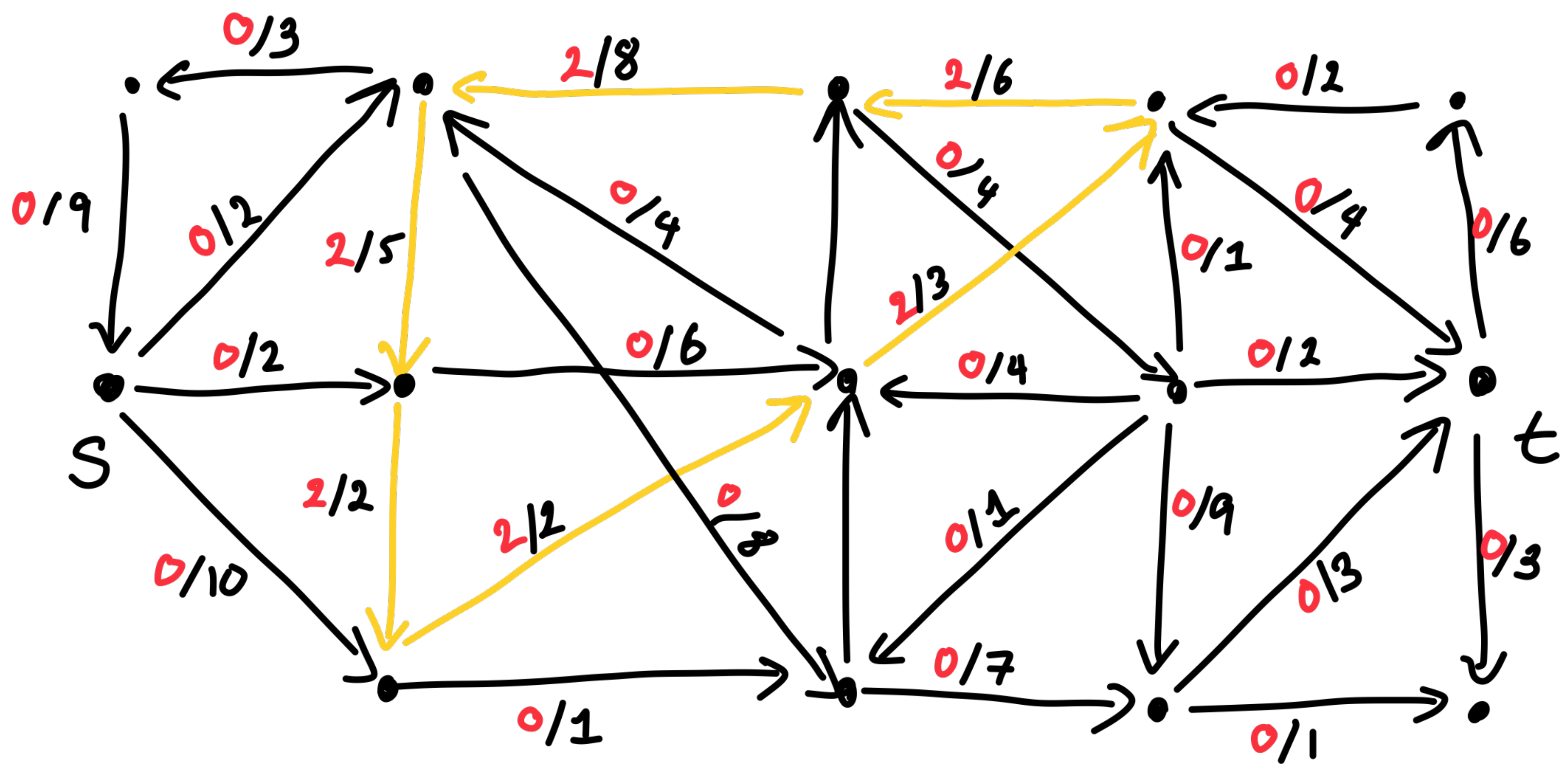
Network connectivity

- Network connectivity and edge disjoint paths use the same reduction
  - Network connectivity is equivalent to min cut
  - Edge disjoint paths is equivalent to max flow
- **Menger's theorem**: the maximum number of edge disjoint s-t paths is equal to the minimum size of a disconnecting set



Directed flow cycle

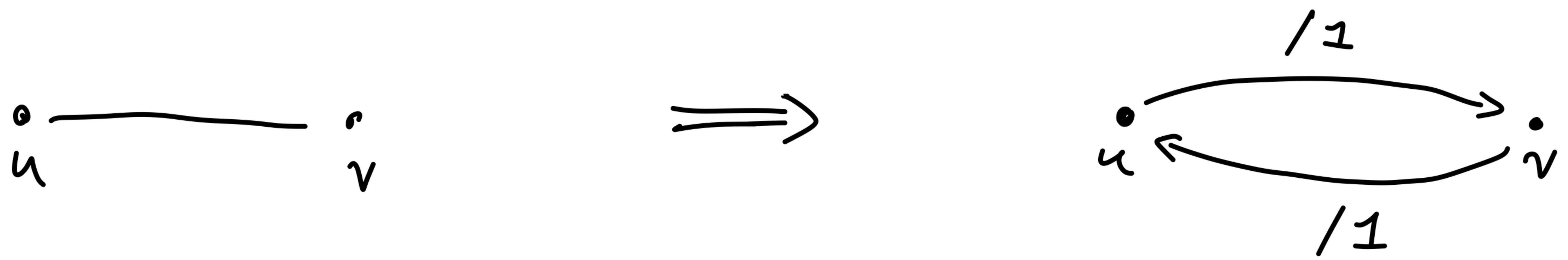
- **Definition:** A directed flow cycle is a flow of value 0 but $f \not\equiv 0$ on every edge
- **Examples:**



- Directed flow cycles can be removed by running graph traversal on f, finding cycles and removing bottleneck flow around the cycle

Undirected graphs

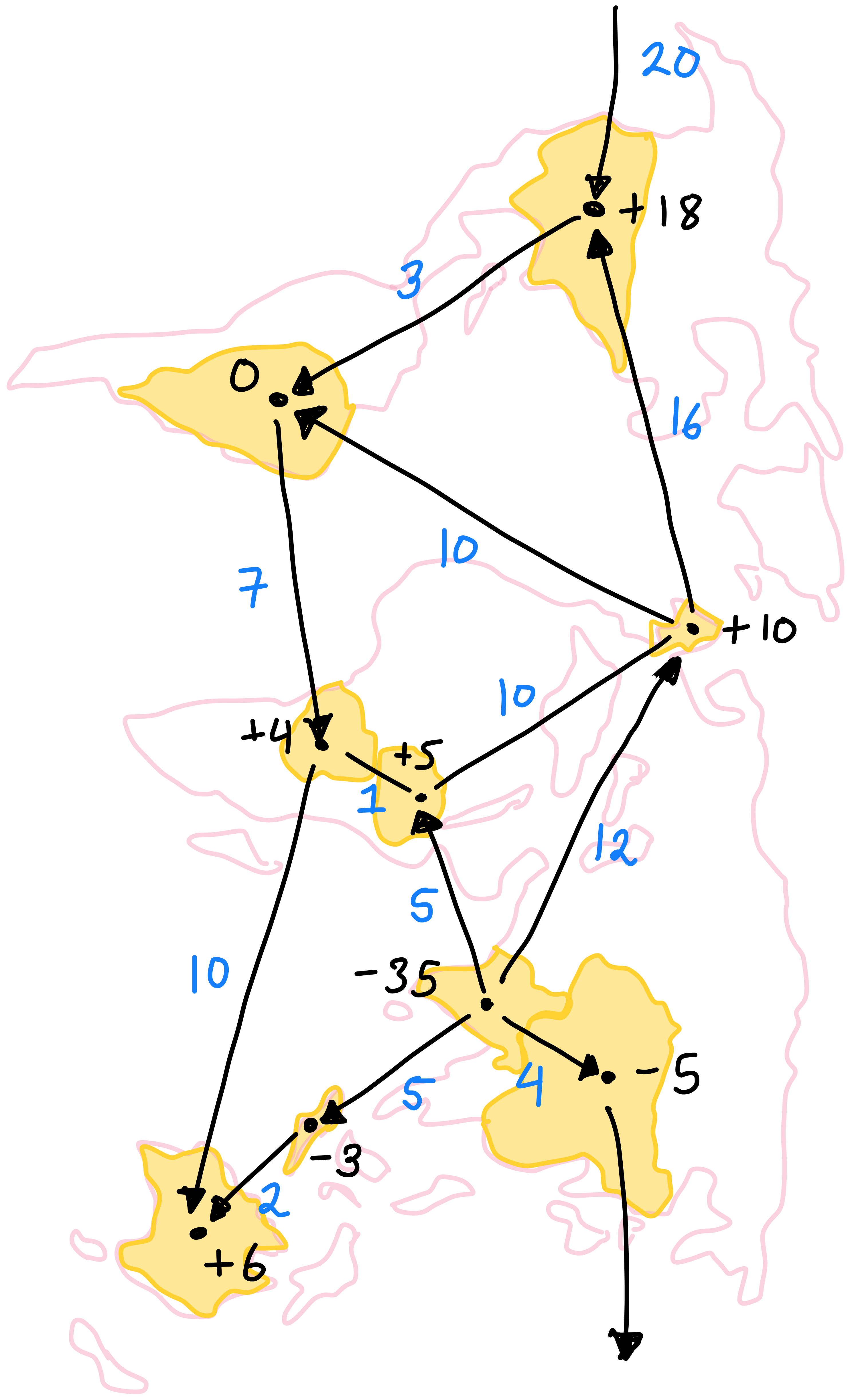
- Edge disjoint path and disconnecting set problems can be solved with flow algorithms for *directed* graphs
- What about undirected graphs?
- **Solution:** Replace each edge (u, v) with directed edges $(u \to v), (v \to u)$



  - Run directed algorithm on new graph
  - Remove any directed flow cycles
  - Include edge $\{u, v\}$ if either edge is used after removing flow cycles

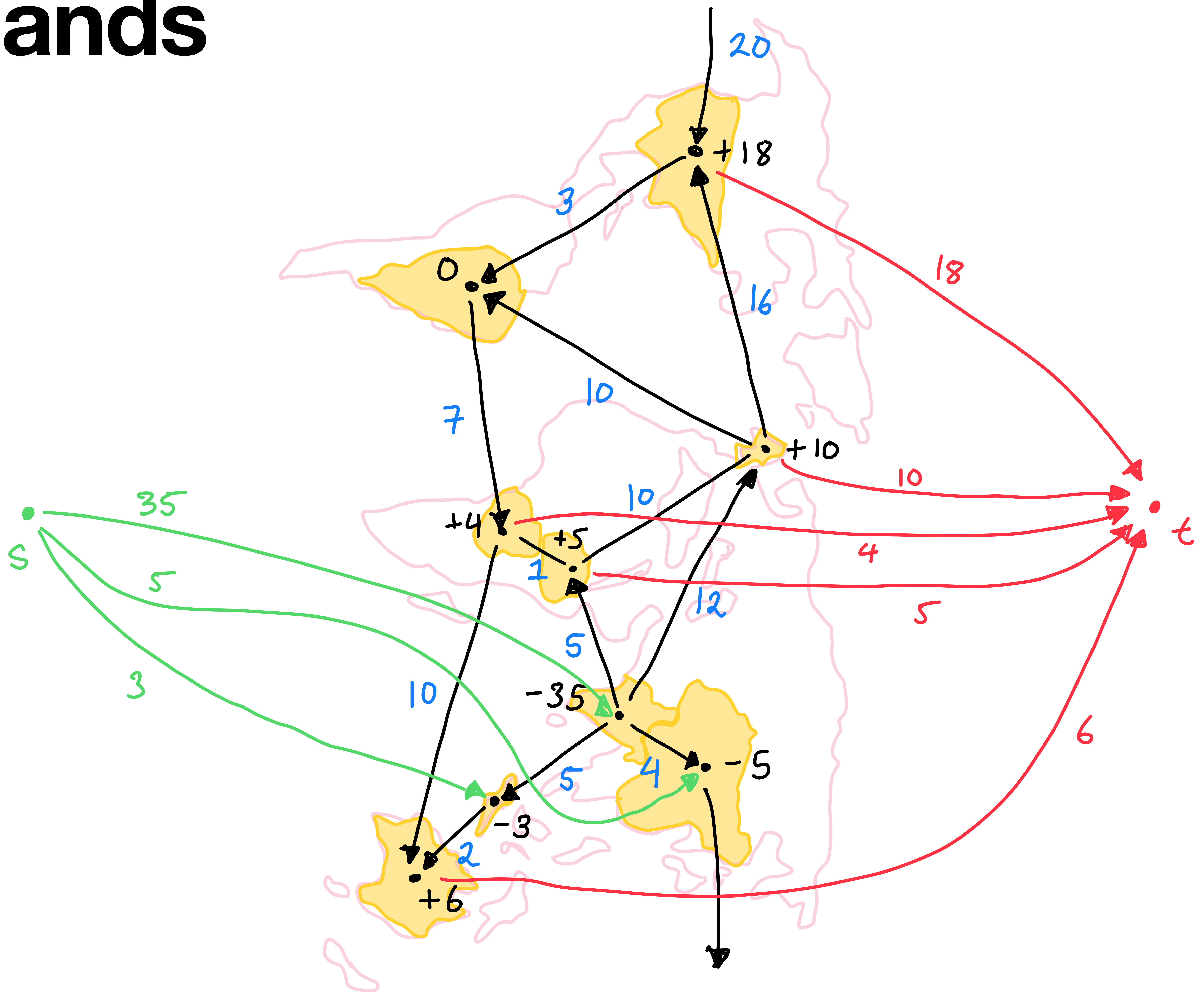
Circulation Demands

- Some countries produce more rice than the consume and some countries consume more rice than the consume
- There are trade routes that describe which countries can trade with which others and at what capacity
- How do we calculate rice routing?
- **Input:** directed graph $G = (V, E)$ with capacities $c : E \to \mathbb{R}_{\geq 0}$ and demand $d : V \to \mathbb{R}$ such that $\sum_{v \in V} d(v) = 0.$
- **Output:** A flow $f : E \to \mathbb{R}$ such that $f^{\text{in}}(v) - f^{\text{out}}(v) = d(v)$



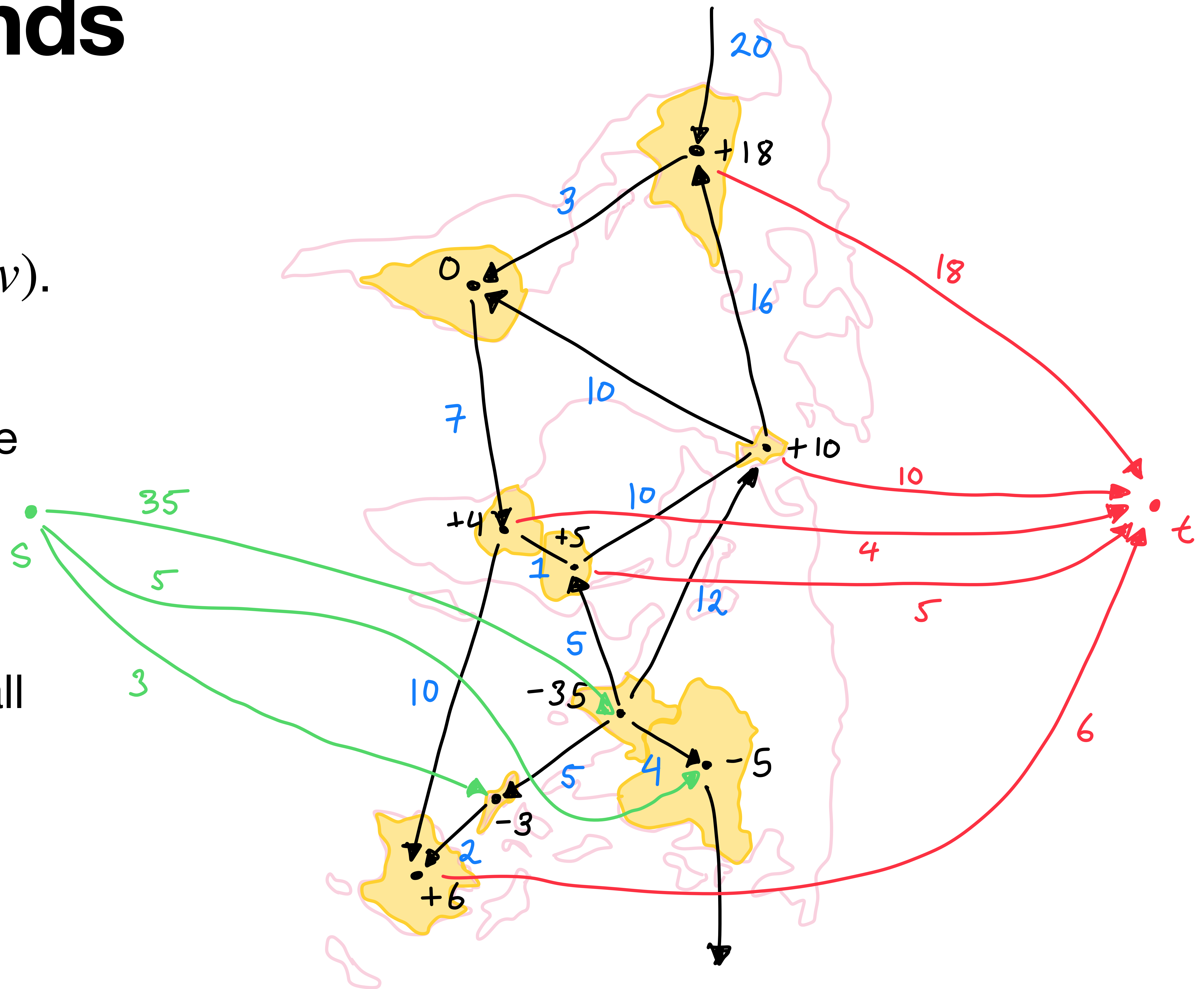
Circulation demands

- Add source s and t to graph
- Add edge $s \rightarrow v$ of $-d(v)$ if $d(v) < 0$.
- Add edge $v \rightarrow t$ of $d(v)$ if $d(v) \geq 0$.
- Compute max flow on the graph.



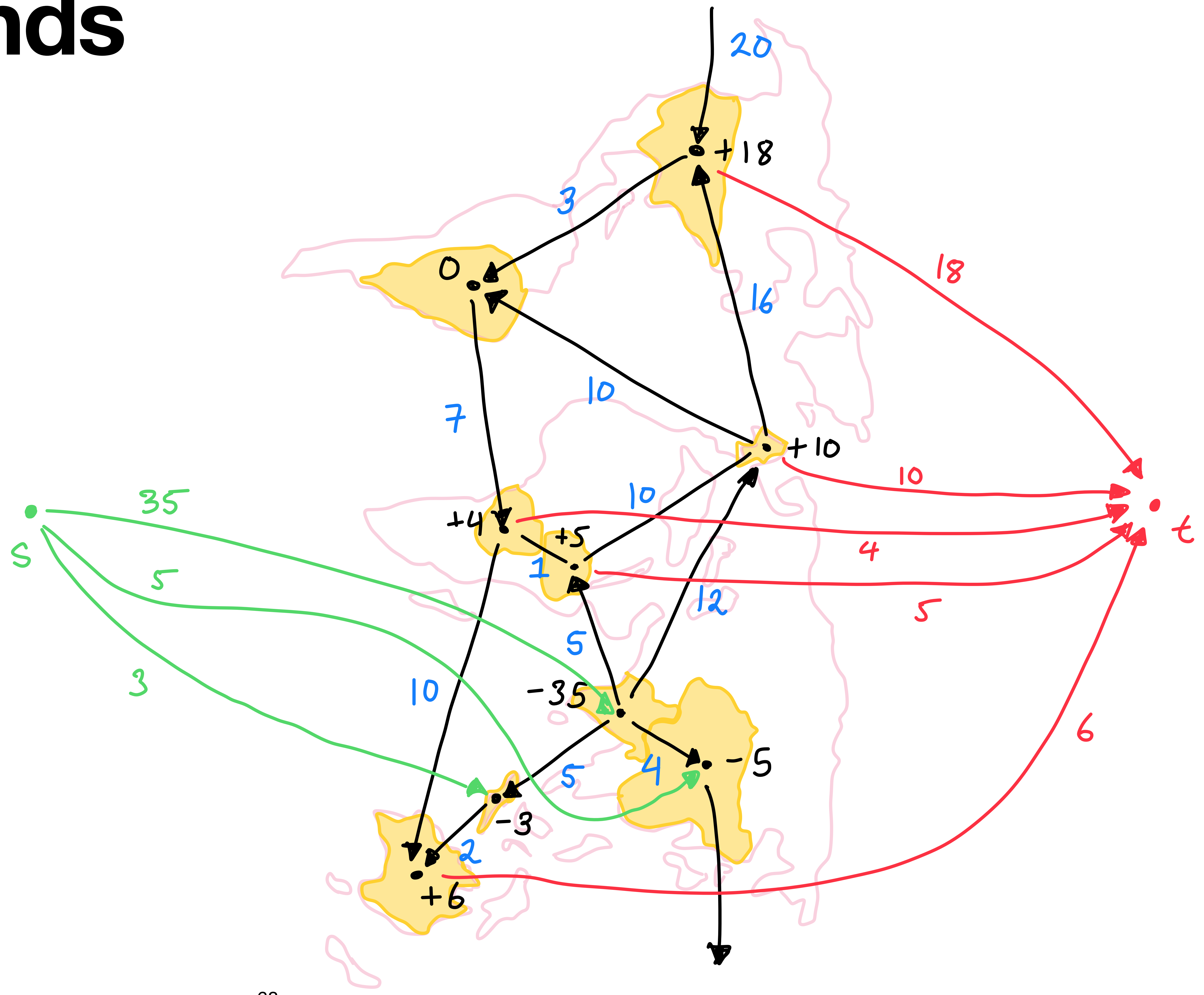
Capacity demands

- **Theorem:** Let $D = \sum_{v:d(v) \geq 0} d(v)$.
  - Then if, max flow = D, there is a *circulation* meeting all capacities and demands.
  - If max flow $< D$, then no circulation exists meeting all capacities and demands. $D - v(f)$ is the "wasted" production.



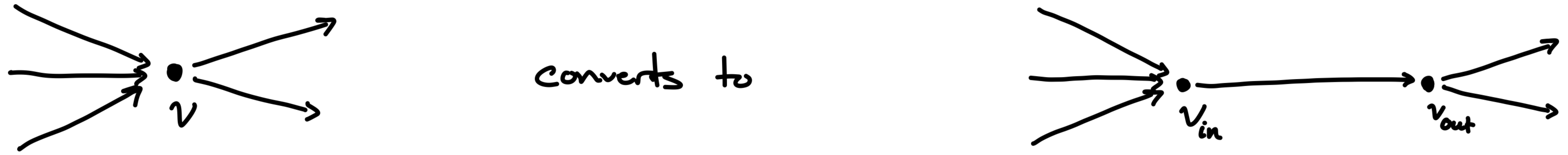
Capacity demands

- When does a circulation not exist? When max flow = min cut $< D$.
- Min-cut between "source" and "sink" vertices is smaller than demand.
- Look at India: The trade network is too small to satisfy the output.



General max flow/min cut algorithmic paradigm

- If source and sink are not obvious, they may need to be added to the graph
- We need to choose capacity limits for edges: 0, 1, ∞ or an input from the problem are logical choices
- Undirected graphs will need to be converted to directed equivalents
  - Unnecessary flow cycles can be removed after flow is calculated
- Split a vertex into two (will show up on problem set):



- Choose correct version of flow algorithm based on capacities

Cut like problems

- Until now, most of the problems looked mostly “flow”-like
- Max flow = min cut tells us that there are probably many “cut”-like problems we can also solve
- Next: an examples of a cut-like problem
  - Goal here is to get you to see flow networks appear in unexpected situations
  - This is at the heart of learning algorithms

Baseball winner

- Imagine a simplified scenario — the team(s) that wins the most games overall is crowned the winner(s).
- Midway through the season, we have the following win totals for the teams

Team	Wins	Games remaining vs Angels	Games remaining vs Rangers	Games remaining vs Athletics	Games remaining vs Mariners
Angels	81	—	5	4	3
Rangers	80	5	—	3	4
Athletics	69	4	3	—	5
Mariners	70	3	4	5	—

Could the Mariners possibly win or tie for first?

Baseball winner

- Best case is Mariners win out — 82 wins
- Still depends on how the other teams play each other. How do we algorithmically calculate this?
- In order to win, Mariners must have a run total at least as high as every other team.

Team	Wins	Games remaining vs Angels	Games remaining vs Rangers	Games remaining vs Athletics	Games remaining vs Mariners
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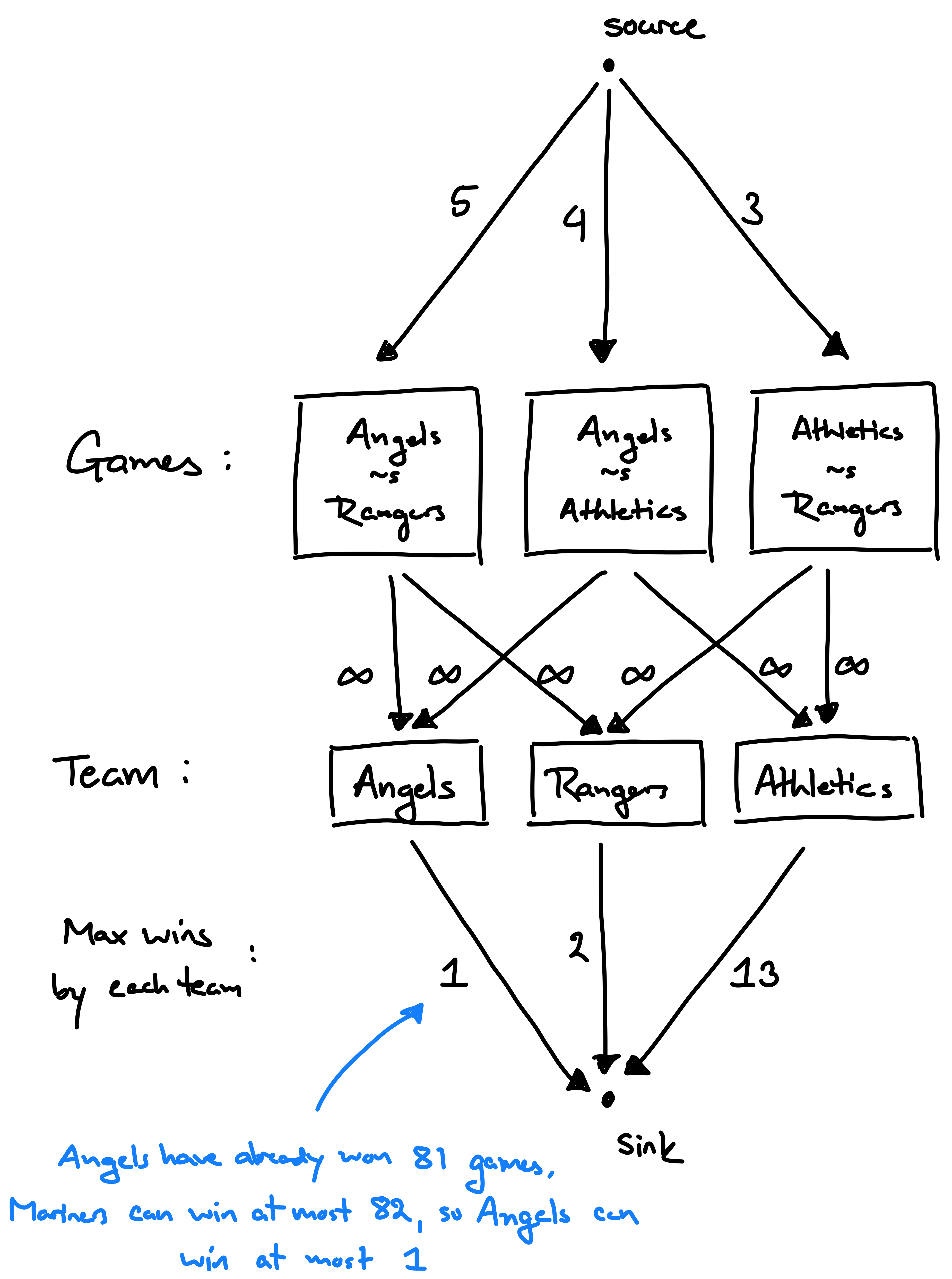

Baseball winner

- If there was a way that the games could play out such that no team amassed > 82 wins then there would be a flow of value $5 + 4 + 3 = 12$ in this network.

- However, the min cut equals = 6

Team	Wins	Games remaining vs Angels	Games remaining vs Rangers	Games remaining vs Athletics	Games remaining vs Mariners
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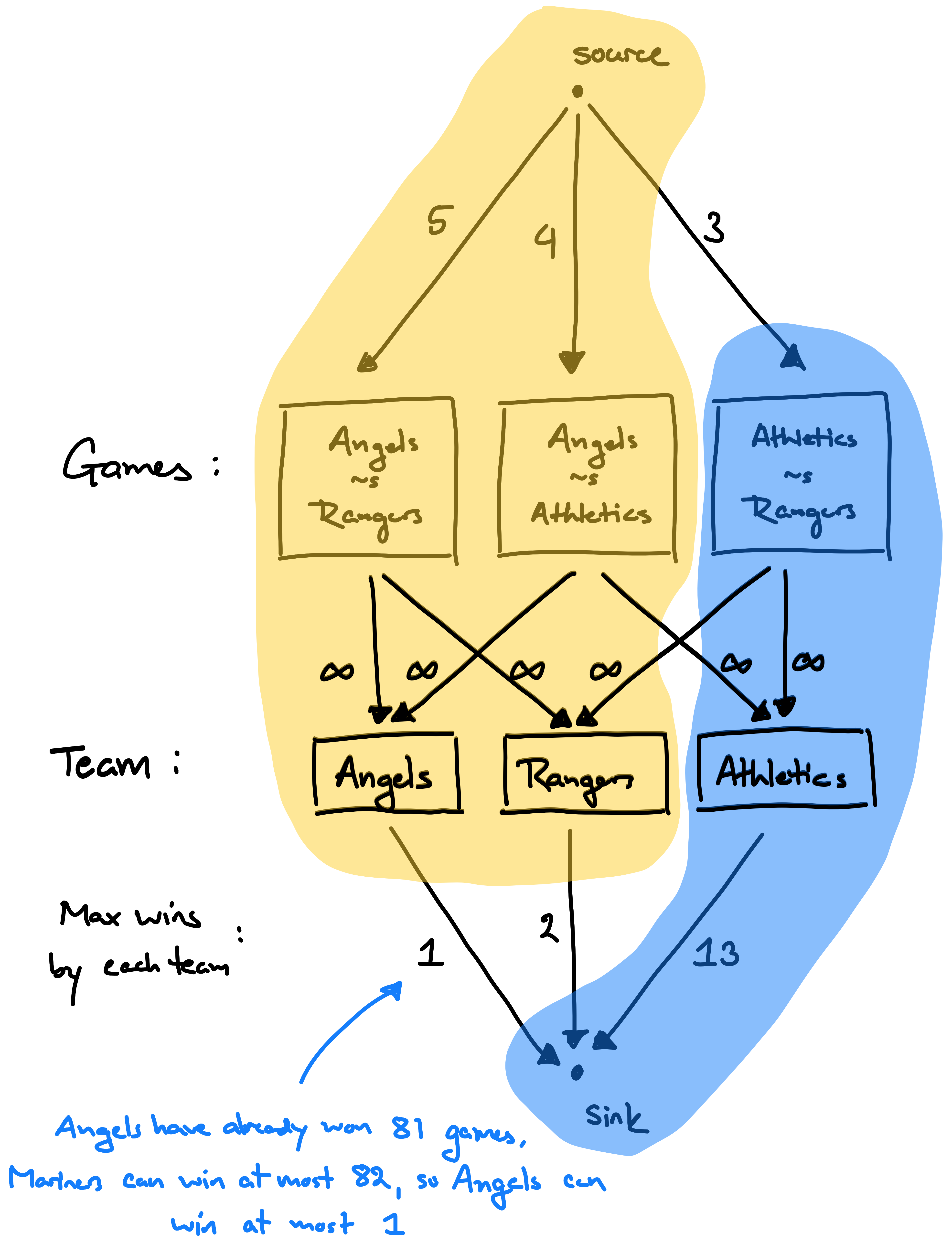

Baseball winner

- Even though no team has won > 82 games yet, this mathematically proves that the Mariners cannot win/tie for 1st.
- A clever way to consider all possible scenarios without exploring all the remaining games.

Team	Wins	Games remaining vs Angels	Games remaining vs Rangers	Games remaining vs Athletics	Games remaining vs Mariners
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Rangers	80	5	—	3	4
Athletics	69	4	3	—	5
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