CSE 421 Section 9

Min-Cut & NP Intro
Administrivia
Announcements & Reminders

- HW8
  - Due Friday!
Task Selection
Task Selection

- Precedence graph $G=(V,E)$
- Each $v$ in $V$ has a profit $p(v)$
- A set $F$ is feasible if when $w$ in $F$, and $(v,w)$ in $E$, then $v$ in $F$.
- Find a feasible set to maximize the profit
Task Selection

- Try to compute by yourself.
- How do we transfer it into Min-Cut problem?
Precedence graph construction

- 1. How to build the edge?
- 2. How to set the edge costs?
- 3. How to make sure it is feasible?
How to build the edge?

- Add vertices s, t
- Each vertex in V is attached to s and t with finite capacity edges
How to set the edge costs?

- If \( p(v) > 0 \),
  - \( \text{cap}(v,t) = p(v) \)
  - \( \text{cap}(s,v) = 0 \)
- If \( p(v) < 0 \)
  - \( \text{cap}(s,v) = -p(v) \)
  - \( \text{cap}(v,t) = 0 \)
- If \( p(v) = 0 \)
  - \( \text{cap}(s,v) = 0 \)
  - \( \text{cap}(v,t) = 0 \)
How to make sure it is feasible?

- Each edge in E has infinite capacity.
- Why?

- The sink side of a finite cut is a feasible set.
- No edges permitted from S to T
- If a vertex is in T, all of its ancestors are in T
Why Min-Cut gives optimal solution?

- Cost(W) = $\sum_{\{w \in W; p(w) < 0\}} -p(w)$
- Benefit(W) = $\sum_{\{w \in W; p(w) > 0\}} p(w)$
- Profit(W) = Benefit(W) – Cost(W)

- Maximum cost and benefit
  - C = Cost(V)
  - B = Benefit(V)
Express Cap(S,T)

- \( \text{Cap}(S,T) = \text{Cost}(T) + \text{Ben}(S) = \text{Cost}(T) + \text{Ben}(S) + \text{Ben}(T) - \text{Ben}(T) \)
- \[ = B + \text{Cost}(T) - \text{Ben}(T) = B - \text{Profit}(T) \]
Image Segmentation
Image analysis

- $a_i$: value of assigning pixel $i$ to the foreground
- $b_i$: value of assigning pixel $i$ to the background
- $p_{ij}$: penalty for assigning $i$ to the foreground, $j$ to the background or vice versa
- $A$: foreground, $B$: background
- $Q(A, B) = \sum_{i \in A} a_i + \sum_{j \in B} b_j - \sum_{(i,j) \in E, i \in A, j \in B} p_{ij}$
Mincut Construction
a: [4,9,2,3]
b: [4,7,4,2]
p1: [0,1,1,0]
p2: [2,0,0,3]
p3: [1,0,0,4]
p4: [0,2,1,0]
Best Result

- Choose 2 and 4 for foreground.
P&NP
What is P?

- Decision problems with polynomial time algorithms
What is NP?

- Problems solvable in non-deterministic polynomial time
- Problems where “yes” instances have polynomial time checkable certificates
How to show it is P?

- Is x a multiple of y?
- Is d the minimal distance from S to T?
- Is the edit distance between x and y less than d?
- Is the max profit of feasible set w?
- Division, Mod
- Shortest Path
- Dynamic Programming
- Network Flow
How to show it is NP?

- 3-SAT
- Independent set of size $K$ – The Independent set of size $K$
- $K$-coloring a graph – Assignment of colors to the vertices
3-SAT

- SAT: Does a given CNF formula have a satisfying formula?
- 3-SAT: each clause is limited to exactly three literals.
- The literals within a clause can be either a Boolean variable or its negation, and the clauses are connected by logical AND operators.
- Instance S and certificate T:

\[
(\overline{x_1} \lor x_2 \lor x_3) \land (x_1 \lor \overline{x_2} \lor x_3) \land (x_1 \lor x_2 \lor x_4) \land (x_1 \lor x_3 \lor x_4)
\]

\[x_1 = 1, \ x_2 = 1, \ x_3 = 0, \ x_4 = 1\]
How to check the certificate T?
How to check the certificate $T$?

- Take given values into Boolean expression. Only takes $O(n)$.
Independent set

- A set of vertices in a graph, no two of which are adjacent.
How to check the given independent set?
How to check the given independent set?

- Go through all edges \((u,v)\). Check whether \(u\) and \(v\) are both in the given independent set. \(O(m)\).
K-coloring

- Give all vertices a color (No more than K different colors). No two adjacent vertices are of the same color.
How to check
How to check

- Go through all edges \((u,v)\). Check whether \(u\) and \(v\) are in the same color. \(O(m)\).
That’s All, Folks!

Thanks for coming to section this week! Any questions?