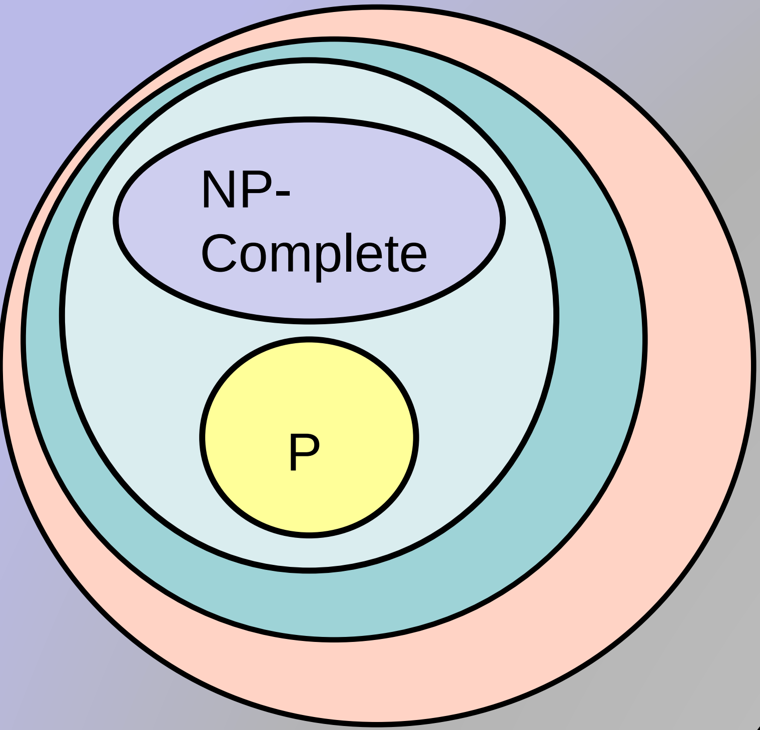
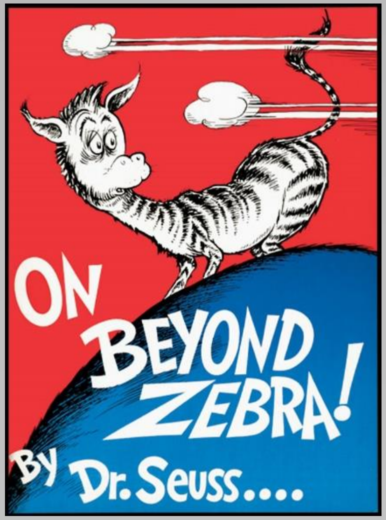




CSE 421
Introduction to Algorithms

Winter 2024

Lecture 26

NP-Completeness and Beyond

Announcements

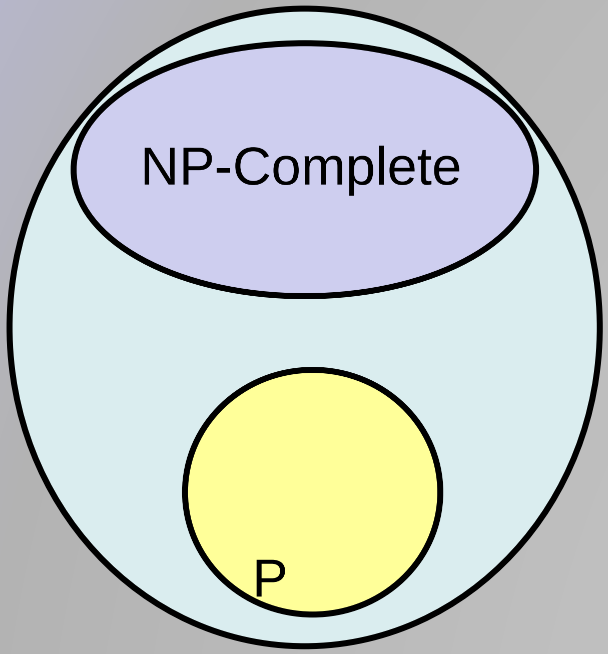
- Final Exam: Monday, March 11, 2:30-4:20 PM
  - One Hour Fifty Minutes
  - Comprehensive (but roughly 60% post midterm)
  - Topics will include: dynamic programming, network flow, network flow reductions, NP-completeness, and other stuff
- Daylight Saving Time starts 2:00 AM, March 10

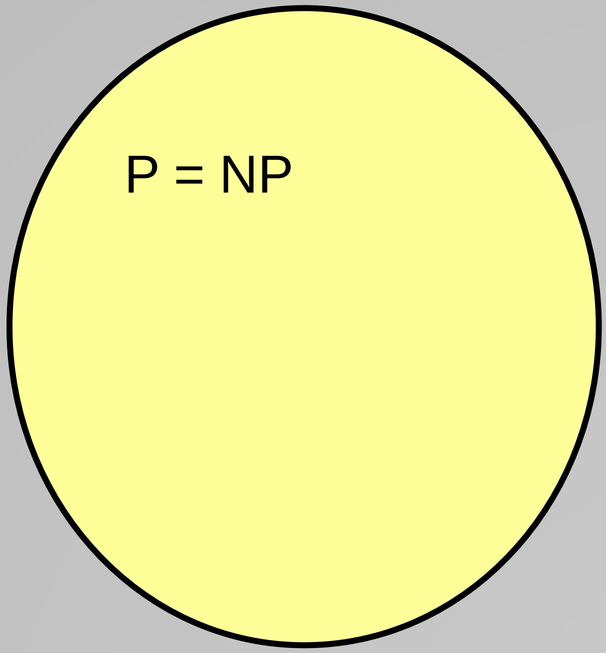
NP-Completeness Proofs

- Prove that problem X is NP-Complete
  - Show that X is in NP (usually easy)
  - Pick a known NP complete problem Y
  - Show $Y <_P X$

What we don’t know

- P vs. NP

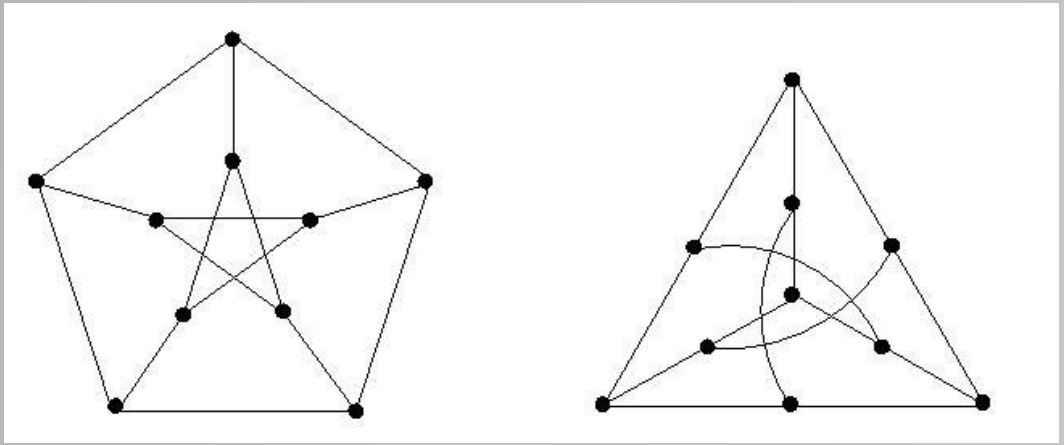




If P $\neq$ NP, is there anything in between

- Yes, Ladner [1975]
- Problems not known to be in P or NP Complete
  - Shortest Vector in a Lattice
  - Factorization
  - Discrete Log
  - Graph Isomorphism

Solve $g^k = b$ over a finite group

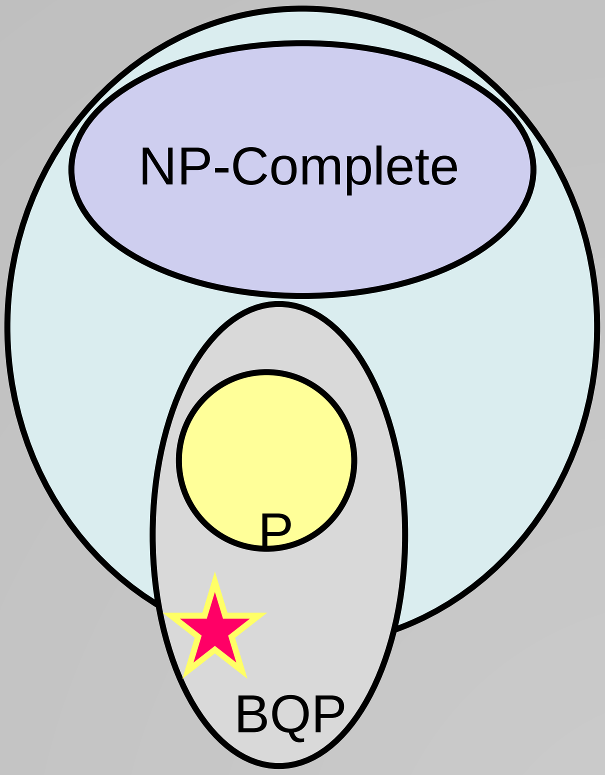


What if?

- 3-SAT can be solved in $O(n^3)$ time
- 3-SAT can be solved in $O(n^{5000})$ time
- Factorization can be solved in $O(n^3)$ time

What about Quantum?

- Computing with Quantum Devices
  - Superposition of states
- Complexity Theory: BQP - Bounded Error Quantum Polynomial Time
- Factorization is in BQP Time (Shor's Algorithm)

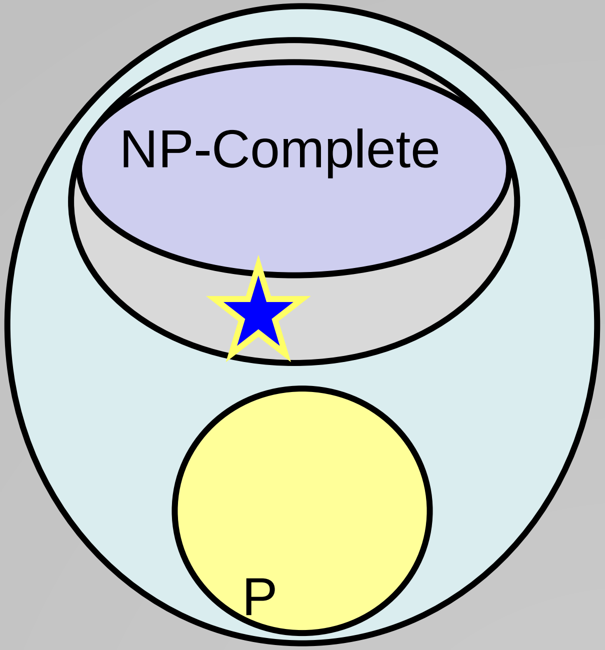


Cryptography

- Standard cryptography depends on number theory problems being hard
  - Keeping factorization secret
- Practical Quantum would break RSA
- Post-Quantum Cryptography
  - Find other hard problems to base cryptography on

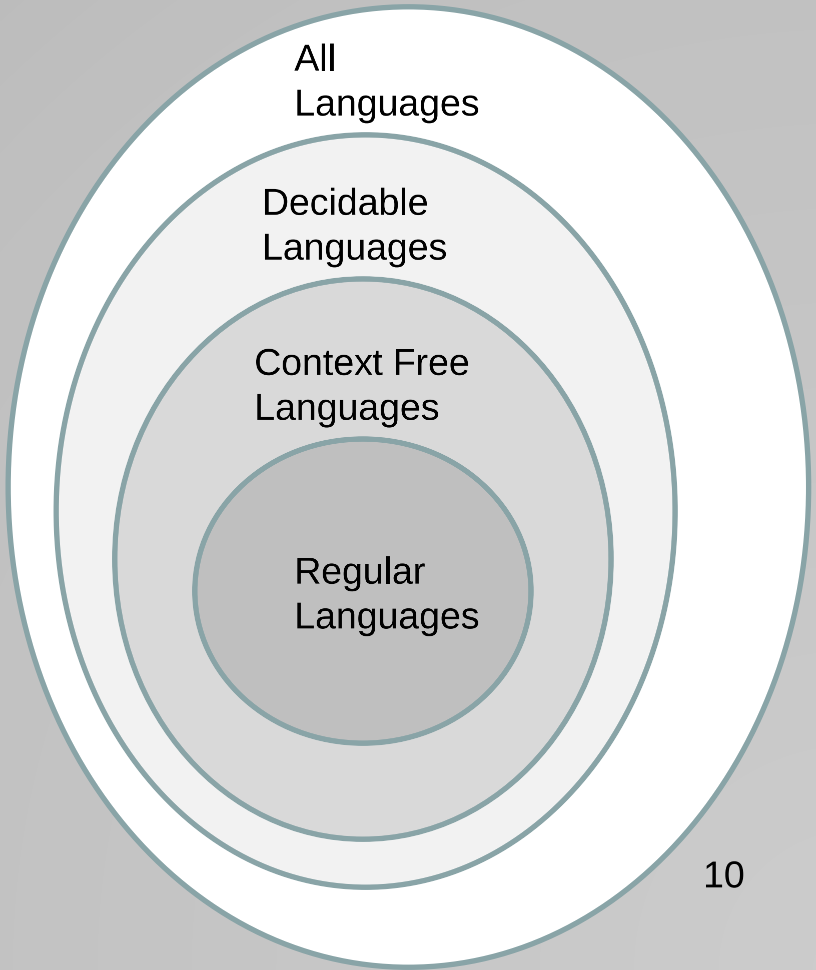
Shortest Vector in a Lattice

- Given a set of vectors L, what is the shortest non-zero vector that can be formed by integer linear combinations of the vectors?
- The problem is NP-Complete under randomized polynomial time reductions



Complexity Theory

- Computational requirements to recognize languages
- Models of Computation
- Resources
- Hierarchies



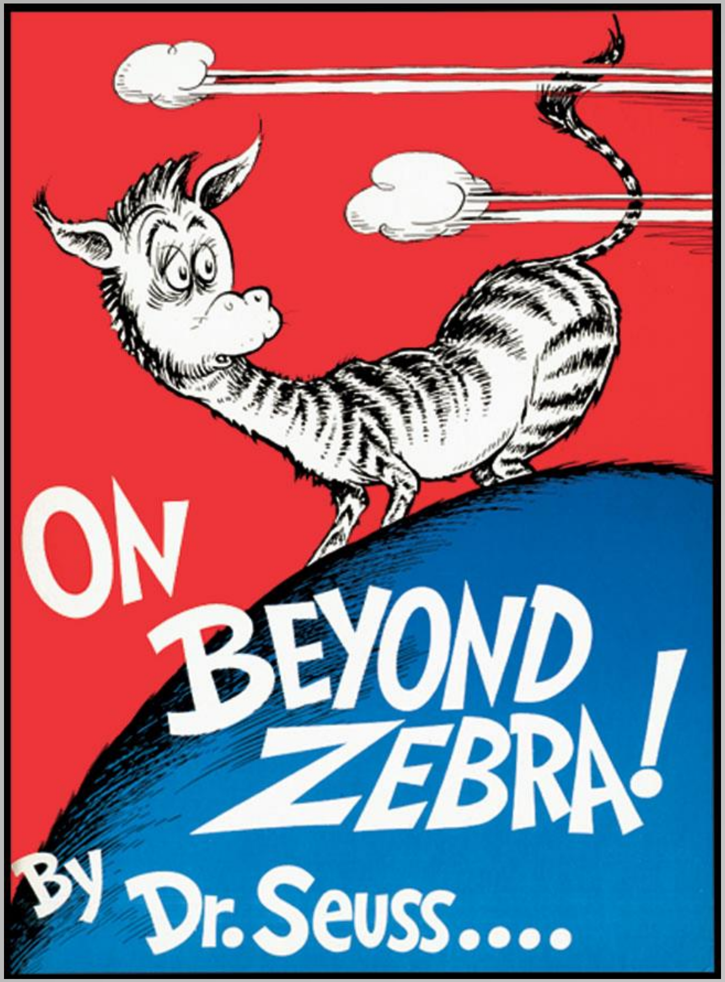
Time complexity

- P: (Deterministic) Polynomial Time
- NP: Non-deterministic Polynomial Time
- EXP: Exponential Time

Space Complexity

- Amount of Space (Exclusive of Input)
- L: Logspace, problems that can be solved in O(log n) space for input of size n
  - Related to Parallel Complexity
- PSPACE, problems that can be required in a polynomial amount of space

So what is beyond NP?



NP vs. Co-NP

- Given a Boolean formula, is it true for some choice of inputs

- Given a Boolean formula, is it true for all choices of inputs

Problems beyond NP

- Exact TSP, Given a graph with edge lengths and an integer K, does the minimum tour have length K

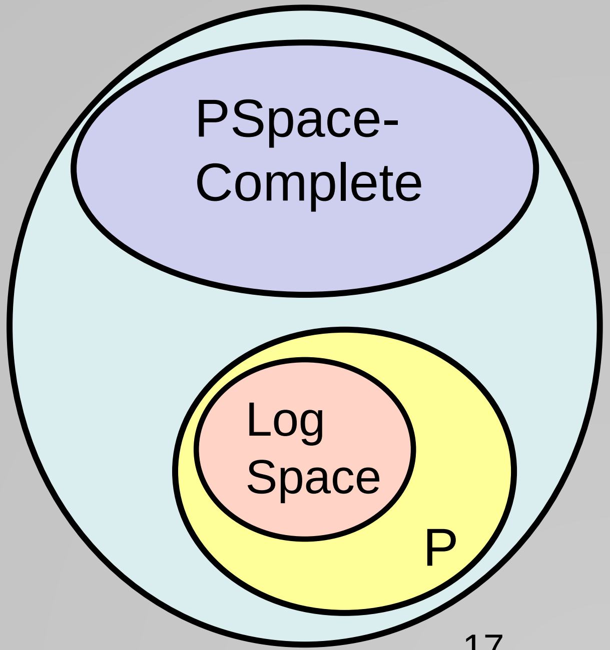
- Minimum circuit, Given a circuit C, is it true that there is no smaller circuit that computes the same function a C

Polynomial Hierarchy

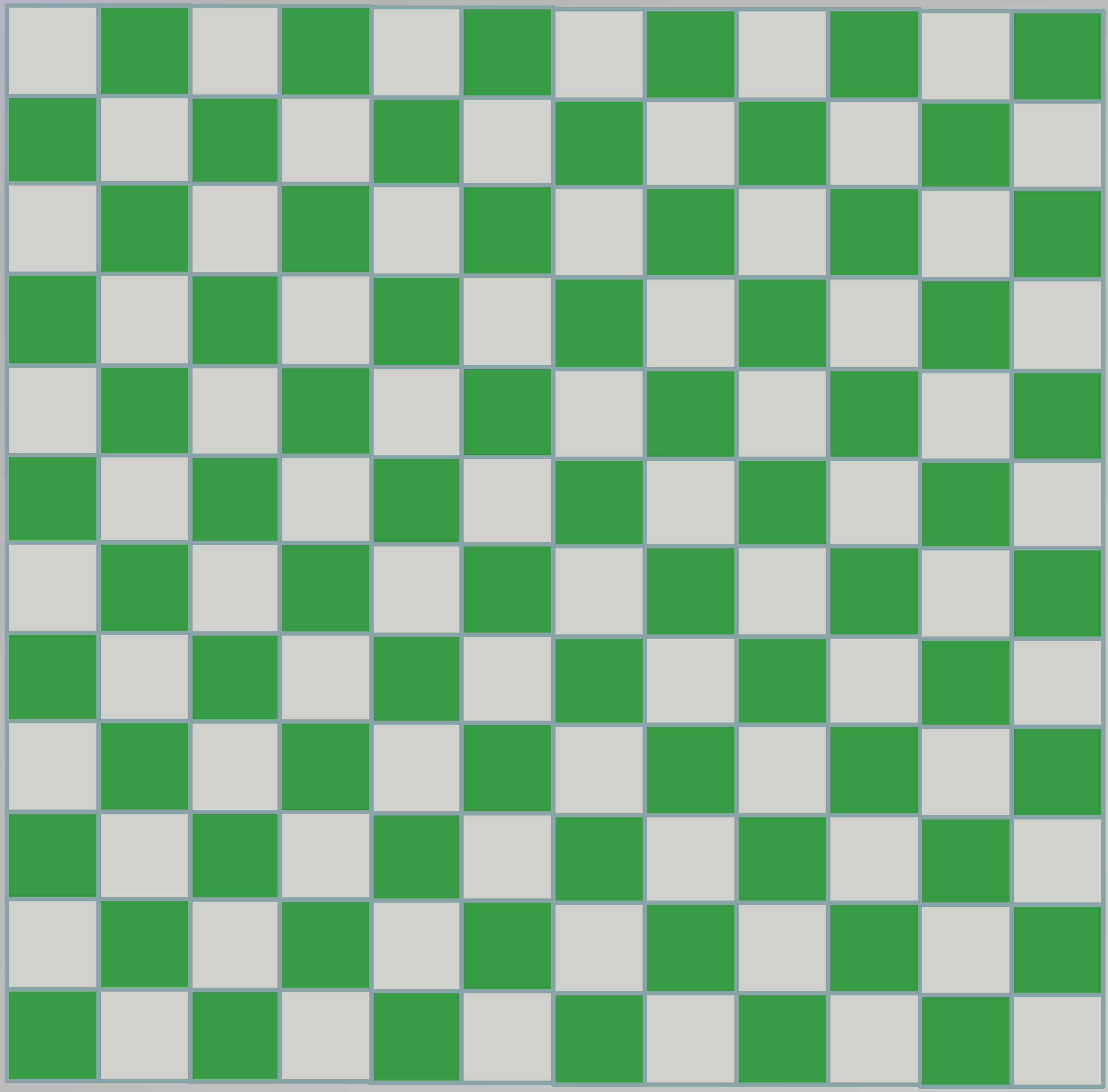
- Level 1
  - $\exists X_1\ \Phi(X_1)$, $\forall X_1\ \Phi(X_1)$
- Level 2
  - $\forall X_1 \exists X_2\ \Phi(X_1,X_2)$, $\exists X_1 \forall X_2\ \Phi(X_1,X_2)$
- Level 3
  - $\forall X_1 \exists X_2 \forall X_3\ \Phi(X_1,X_2,X_3)$, $\exists X_1 \forall X_2 \exists X_3\ \Phi(X_1,X_2,X_3)$

Polynomial Space

- Quantified Boolean Expressions
  - $\exists X_1 \forall X_2 \exists X_3 \ldots \exists X_{n-1} \forall X_n \; \Phi(X_1, X_2, X_3 \ldots X_{n-1} X_n)$
- Space bounded games
  - Competitive Facility Location Problem
  - N x N Chess
- Counting problems
  - The number of Hamiltonian Circuits



N X N Chess



Even Harder Problems

```
public int[] RecolorSwap(int[] coloring) {
        int k = maxColor(coloring);

        for (int v = 0; v < nVertices; v++) {
            if (coloring[v] == k) {
                int[] nbdColorCount = ColorCount(v, k, coloring);
                List<Edge> edges1 = vertices[v].Edges;

                foreach (Edge e1 in edges1) {
                    int w = e1.Head;
                    if (nbdColorCount[coloring[w]] == 1)
                        if (RecolorSwap(v, w, k, coloring))
                            break;
                }
            }
        }
        return coloring;
    }
```

Is this code correct?

Halting Problem

- Given a program P that does not take any inputs, does P eventually exit?

Impossibility of solving the Halting Problem

Suppose Halt(P) returns true if P halts, and false otherwise

Consider the program G:

What is Halt(G)?

```
Define G {
    if (Halt(G)){
        while (true) ;
    }
    else {
        exit();
    }
}
```

Undecidable Problems

- The Halting Problem is undecidable
- Impossible problems are hard . . .