University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2019

January 31, 2024

Midterm Exam, Wednesday, October 30, 2019

NAME: ______________________

Instructions:

- Closed book, closed notes, no calculators
- Time limit: 50 minutes
- Answer the problems on the exam paper.
- If you need extra space use the back of a page
- Problems are not of equal difficulty, if you get stuck on a problem, move on.

1	/15
2	/10
3	/10
4	/10
5	/5
6	/15
Total	/65

Problem 1 Graph Theory (15 points):

a) *True or false*: Let $G = (V, E)$ be an undirected graph. If G is a tree, then G is bipartite. Justify your answer.

b) *True or false*: Let $G = (V, E)$ be an undirected graph with n vertices and m edges. If $m > \frac{n(n-1)}{2} - (n-1)$ then the graph G must be connected. (By definition, an undirected graph has no parallel edges and no self loops.) Justify your answer.

c) *True or false*: Let $G = (V, E)$ be a directed graph with n vertices. If every vertex of G has out degree at least one, then G has a cycle. Justify your answer.

Problem 2 Space Aliens (10 points):

There is a new alien language which uses the Latin alphabet. However, the order among letters is unknown to you. You receive a list of sequences of letters in alphabetical order and you want to construct a total order consistent with the sequences.

For example, given the following sequences: (a,d,e), (a, c, b), (d, g, e), (d,b,e), your algorithm could return (a, d, c, g, b, e). If there are multiple valid order of letters, simply return any one of them.

Give an algorithm with runs in $O(n+m)$ time, where n is the number of letters, and m is the sum of the lengths of the sequences.

Problem 3 Minimum Spanning Trees (10 points):

Let $G = (V, E)$ be an undirected graph with edge weights. We will assume that the edge weights are distinct.

a) What is the edge inclusion lemma (for Minimum Spanning Trees)?

b) Let u be a vertex, and suppose that $e = (u, v)$ is the minimum cost edge adjacent to u. Prove that the edge e is in the minimum spanning tree for G.

Problem 4 Interval Scheduling (10 points):

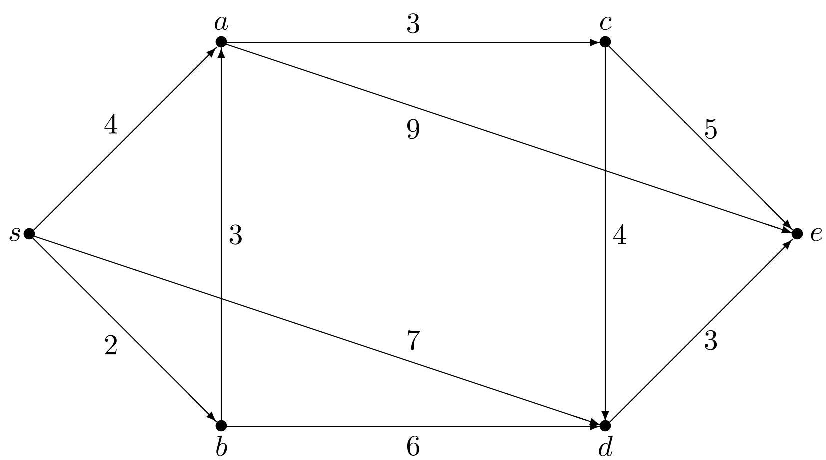
The input for an interval scheduling problem is a set of intervals $I = \{i_1, \ldots, i_n\}$ where i_k has start time s_k, and finish time f_k. The problem is to find a set of non-overlapping intervals that satisfies a given criteria.

a) Suppose that you want to maximize the total length of the selected intervals. *True or false*: The greedy algorithm based on selecting intervals in order of decreasing length finds an optimal solution. Justify your answer.

b) The set of intervals $I' = \{i'_1, \ldots, i'_n\}$ is said to be a *shrinking* of the intervals $I'' = \{i''_1, \ldots, i''_n\}$ if each interval in I' is contained in the corresponding interval of I'', in other words, for $1 \leq k \leq n$, $s''_k \leq s'_k \leq f'_k \leq f''_k$. *True or false*: If I' is a shrinking of I'', then the maximum number of non-overlapping intervals in I' is at least as great as the maximum number of non-overlapping intervals in I''. Justify your answer.

Problem 5 Bottleneck Distance (5 points):

The bottleneck length of a path P is the cost of the maximum edge of P. The bottleneck distance between vertices u and v is the minimum bottleneck distance of a path between u and v. This problem can be solved using a variant of Dijkstra's algorithm.



Simulate Dijkstra's bottleneck path algorithm on the graph above by filling in the table. The entries should contain the preliminary distance values.

Round	Vertex	s	a	b	c	d	e
1							
2							
3							
4							
5							
6							

Problem 6 Recurrences (15 points):

Give solutions to the following recurrences. Justify your answers.

a)

$$T(n) = \begin{cases} 5T(\frac{n}{3}) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

b)

$$T(n) = \begin{cases} T(\frac{4n}{5}) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$

c)

$$T(n) = \begin{cases} 16T(\frac{n}{4}) + n^2 & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$$