CSE421: Design and Analysis of Algorithms May 23, 2024

Shayan Oveis Gharan Problem Solving Session 8

P1) Write the following LP in Standard Form:

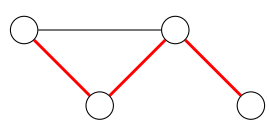
$$\begin{aligned} \min \quad & 2x_1 - x_2 \\ \text{s.t.,} \quad & x_1 - x_3 = 4 \\ & 2x_2 - x_3 \geq 5 \\ & x_1, x_3 \geq 0. \end{aligned}$$

P2) Write the dual of the following program:

$$\begin{aligned} \max \quad & 2x_1 + x_2 \\ \text{s.t.,} \quad & x_1 + x_2 \leq 5 \\ & x_1 - 2x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

P3) Write the dual of the LP relaxation of the min vertex cover then turn it into the standard form.

P4) A Hamiltonian cycle in a directed graph with n vertices is a directed cycle of length n, i.e., it is a cycle that visits all vertices exactly once and returns back to the starting point. A directed Hamiltonian path in a graph with n vertices is a path of length $n-1$, i.e., it is a path that visits all vertices of the graph exactly once. For example, the following graph has a Hamiltonian path marked in red but no Hamiltonian cycle.



Hamiltonian-cycle problem is defined as follows: Given a graph $G = (V, E)$, does it have a Hamiltonian cycle?

Hamiltonian-path problem is defined as follows: Given a graph $G = (V, E)$, does it have a Hamiltonian path?

Prove that Hamil-path $\leq_P$ Hamil-cycle. In other words, suppose we have a program A that solves the Hamiltonian cycle problem. Design a polynomial time algorithm that on an input graph G uses A only polynomial number of times and in polynomial time returns the solution to the Hamiltonian path problem on the given graph G.