P1) Write the following LP in Standard Form:

\[
\begin{align*}
\text{min} & \quad 2x_1 - x_2 \\
\text{s.t.,} & \quad x_1 - x_3 = 4 \\
& \quad 2x_2 - x_3 \geq 5 \\
& \quad x_1, x_3 \geq 0.
\end{align*}
\]

P2) Write the dual of the following program:

\[
\begin{align*}
\text{max} & \quad 2x_1 + x_2 \\
\text{s.t.,} & \quad x_1 + x_2 \leq 5 \\
& \quad x_1 - 2x_2 \leq 2 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

P3) Write the dual of the LP relaxation of the min vertex cover then turn it into the standard form.

P4) A Hamiltonian cycle in a directed graph with \( n \) vertices is a directed cycle of length \( n \), i.e., it is a cycle that visits all vertices exactly once and returns back to the starting point. A directed Hamiltonian path in a graph with \( n \) vertices is a path of length \( n - 1 \), i.e., it is a path that visits all vertices of the graph exactly once. For example, the following graph has a Hamiltonian path marked in red but no Hamiltonian cycle.

![Graph with Hamiltonian path marked in red]

Hamiltonian-cycle problem is defined as follows: Given a graph \( G = (V, E) \), does it have a Hamiltonian cycle?

Hamiltonian-path problem is defined as follows: Given a graph \( G = (V, E) \), does it have a Hamiltonian path?

Prove that Hamil-path \( \leq_P \) Hamil-cycle. In other words, suppose we have a program \( A \) that solves the Hamiltonian cycle problem. Design a polynomial time algorithm that on an input graph \( G \) uses \( A \) only polynomial number of times and in polynomial time returns the solution to the Hamiltonian path problem on the given graph \( G \).