P1) Let G be a connected undirected graph let T be the DFS tree with root s. Prove that for any edge $e = (u, v) \in G$, e is in a cycle in G iff one of the following holds:

- e is a non-tree edge,

- e is a tree edge (say u is parent of v) and there is a non-tree edge from a descendent of v to an ancestor of u.

P2) Let G be a graph with n vertices such that the degree of every vertex of G is at most k. Prove that we can color vertices of G with $k + 1$ colors such that the endpoints of every edge get two distinct colors.

Turn your proof into a polynomial time algorithm to color vertices of G with k colors.

P3) Prove or disprove: Every directed graph with n vertices and at least $n(n-1)/2 + 1$ directed edges has a cycle.