CSE421: Design and Analysis of Algorithms	April 4, 2024
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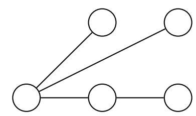
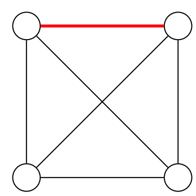
P1) Let G be a tree. Use induction to prove that the number of leaves of G is at least the number of vertices of degree at least 3 in G. For example, the following tree has 3 leaves and 1 vertex of degree at least 3, and $3 \geq 1$.



P2) Let G be a graph with n vertices and at least n edges. Show that G has a cycle.

P3) Given a connected undirected graph $G = (V, E)$ with n vertices and m edges. Design an $O(m + n)$ time algorithm that outputs an edge e of G such that if we delete e, G remains connected. If no such edge exists output “Impossible”. For example in the following graph if you delete the red edges the graph remains connected.



We write the psueodo-code below, although the above description is already enough:

```
Function BFS(s)
    Initialize: mark all vertices "undiscovered"
    mark s "discovered"
    queue = { s }
    while queue not empty do
        u = remove_first(queue)
        for each edge {u, x} do
            if x is "undiscovered" then
                mark x "discovered"
                append x on queue
            end
            else
                output {u, x} and end the algorithm
            end
        end
        mark u "fully-explored"
    end
    output "Impossible"
```

Algorithm 1: Algorithm for P3