P1) Let $G$ be a graph such that $\text{deg}(v) \leq k$ for all $v$. A set $S \subseteq V$ of vertices of $G$ form an independent set if there is no edges between vertices of $S$, i.e., for any $u, v \in S$, $u, v$ are not connected by an edge. Design a polynomial time $O(k)$ approximation algorithm for the maximum independent set problem on $G$. Your algorithm needs to output an independent set $S$ whose size is at least $\Omega(1/k)$ of the maximum independent set of $G$.

P2) Draw the dynamic programming table of the following instance of the knapsack problem: You are given 5 items with weight $1, 3, 5, 7, 9$ and value $1, 2, 4, 5, 7$ respectively and the size of your knapsack is 14.

P3) You are a cashier at a Grocery store in a country with coins of values $v_1, v_2, \ldots, v_n$ dollars (you can assume $v_1, \ldots, v_n$ are positive integers). Furthermore, assume you have an infinite supply of each coin. A customer comes and you need to make a change for $k$ dollars. Design an algorithm that runs in time polynomial in $n, k$ and outputs the minimum number of coins you can use to make a change for $k$ dollars.

P4) Given 3 integers $n \geq 1$ and $k_A, k_B \geq 1$, design an algorithm that runs in time polynomial in $n, k_A, k_B$ and outputs the number of length $n$ strings composed of copies of $A, B$ such that no more than $k_A$ copies of $A$ are placed consecutively and no more than $k_B$ copies of $B$ are placed consecutively. For example, if $n = 3, k_A = k_B = 2$ you should output 6 corresponding to the following sequences:

$$AAB, BBA, ABB, BAA, ABA, BAB$$

P5) Interstate highway 5 is a straight highway from Washington all the way to California. There are $n$ villages alongside this highway. Think about the highway as an integer axis, and the position of village $i$ is an integer $x_i$ along this axis. Assume that there are no two villages in the same position, i.e., $x_i \neq x_j$ for $i \neq j$. The distance between two villages $x_i, x_j$ is simply $|x_i - x_j|$.

USPS is interested in building $k$ post offices in some, but not necessarily all of the villages along highway 5, for some $1 \leq k \leq n$. A village and the post office in it have the same position. We want to choose the positions of these post offices so that the sum of the distances from each village to its nearest post office is minimized. Design an algorithm that runs in time polynomial in $n$ and outputs the minimum possible sum of distances to the optimal location for post offices. For example, given the following location of 5 cities if $k = 2$ then the optimal location for post offices are at village 0 and 4 and your algorithm should output 3 as the sum of distance to nearest post offices.

![Diagram of highway with villages and post offices]
P6) **Extra Credit:** A $k$-hypergraph is composed of a set $V$ of vertices and a set of hyperedges where every hyperedge is a subset of $V$ of size at least 2 and at most $k$, i.e., $S$ is a hyperedge if $S \subseteq V$ and $2 \leq |S| \leq k$. Note that 2-hypergraph is the same as a graph. Given a $k$-hypergraph $G = (V, E)$ with $n$ vertices where for some $k \geq 2$ design a $k$-approximation algorithm for the vertex cover problem: Find the minimum set $W$ of vertices of $G$ such that every hyperedge $S \in E$ has at least one vertex of $W$. 