CSE 421 Section 8

Linear Programming and Technique Toolbox

Administrivia

Announcements & Reminders

- **HW6** was due yesterday, 11/13
  - Late submissions open until tomorrow, 11/15 @ 11:59pm

- **HW7**
  - Due **Friday** 11/22 @ 11:59pm
  - Late submissions will be open until Sunday, 11/24 @ 11:59pm

Linear programming

Review of linear programming

Linear programming is the following problem:

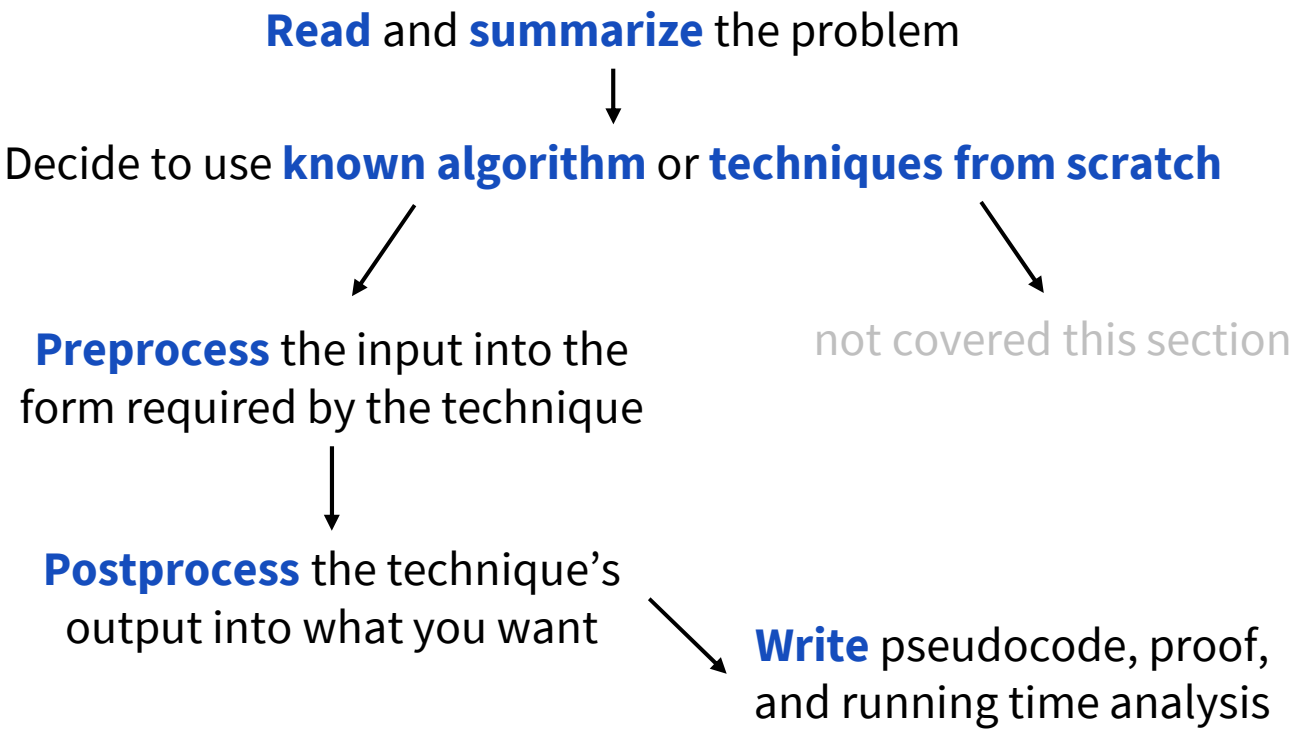
$$\textbf{maximize } c^T x$$
$$\textbf{subject to } Ax \leq b, x \geq 0$$

Standard form is maximization with less than or equal to constraints

In other words,

$$\begin{aligned}
\textbf{maximize } & c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \\
\textbf{subject to } & a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \leq b_1 \\
& a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \leq b_2 \\
& \cdots \\
& a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \leq b_m \\
& x_i \geq 0 \qquad \text{for all } i
\end{aligned}$$

Problem solving strategy overview



Problem 1 – Cost-effective eating

You are given a list of foods indexed $1, \ldots, n$, as well as the calories c_i, sugars (g) s_i, and vitamin D (mcg) d_i per serving of each food. You're trying to maintain a healthy diet by eating exactly 2000 calories per day. You also heard that the American Heart Association recommends at most 30 grams of sugar per day. And because you just moved to Seattle from LA this year, it's your first winter and you need to eat at least 15 mcg of vitamin D to avoid SAD. Along with the nutrition information, you also know that one serving of food i costs m_i money. Find a way to compute a healthy diet that is as cheap as possible.

a) Write a summary of the problem.

Work on this with the people around you, then we'll check!

Solution

Problem 1 – Cost-effective eating

a) Write a summary of the problem.

Input: Calories c_i, sugar s_i, vitamin D d_i, and money m_i per serving for each food.
Expected output: Minimum amount of money to meet nutrition standards.

Problem 1 – Cost-effective eating

b) To use linear programming:

i. What should the variables x_i represent?

ii. What is the objective function?

iii. What are the constraints (directly translated from the problem)?

Work on this with the people around you, then we'll check!

Solution

Problem 1 – Cost-effective eating

b) To use linear programming:

i. What should the variables x_i represent?

The number of servings of food i to eat.

ii. What is the objective function?

$$\textbf{minimize}\ m_1x_1 + \cdots + m_nx_n$$

iii. What are the constraints (directly translated from the problem)?

$$\begin{aligned}\textbf{subject to}\ \ c_1x_1 + \cdots + c_nx_n &= 2000\\ s_1x_1 + \cdots + s_nx_n &\leq 30\\ d_1x_1 + \cdots + d_nx_n &\geq 15\\ x_i &\geq 0 \qquad \text{for all } i\end{aligned}$$

Problem 1 – Cost-effective eating

iv. How can you transform the problem into standard form?

Work on this with the people around you, then we'll check!

Solution

Problem 1 – Cost-effective eating

iv. How can you transform the problem into standard form?

$$\textbf{maximize} \quad -m_1 x_1 - \cdots - m_n x_n$$

$$\begin{aligned} \textbf{subject to} \quad c_1 x_1 + \cdots + c_n x_n &\leq 2000 \\ -c_1 x_1 - \cdots - c_n x_n &\leq -2000 \\ s_1 x_1 + \cdots + s_n x_n &\leq 30 \\ -d_1 x_1 - \cdots - d_n x_n &\leq -15 \\ x_i &\geq 0 \qquad \text{for all } i \end{aligned}$$

1. Multiply “minimize” objective by -1.
2. Convert equalities into two inequalities.
3. Multiply $\geq$ inequalities by -1.

Problem 1 – Cost-effective eating

For basic LP problems, we will only be looking for a brief sketch of correctness, unless there is something nontrivial beyond directly translating the constraints.

c) Sketch the correctness of your solution.

Work on this with the people around you, then we'll check!

Solution

Problem 1 – Cost-effective eating

For basic LP problems, we will only be looking for a brief sketch of correctness, unless there is something nontrivial beyond directly translating the constraints.

c) Sketch the correctness of your solution.

Because m_i is the price per serving, we pay $m_i x_i$ if we pick x_i servings of food i, so our goal is to minimize $m_1 x_1 + \cdots + m_n x_n$. Similarly, we are given per-serving values for the calories, sugar, and vitamin D of each food, so we have $c_1 x_1 + \cdots + c_n x_n = 2000$, $s_1 x_1 + \cdots + s_n x_n \leq 30$, and $d_1 x_1 + \cdots + d_n x_n \geq 15$. We transformed them to standard form using basic algebra. Relying on an LP algorithm, we output the best $x_1, \ldots, x_n$ as desired.

Technique toolbox

Which technique should I try?

We have covered many techniques for algorithms so far.

Using known algorithms	Developing from scratch
• Stable matching • Graph traversal algorithms • Weighted graph algorithms • Network flows • Linear programming	• Greedy algorithms • Divide and conquer • Dynamic programming

How should you pick which method to try?

Problem solving strategy overview

Read and **summarize** the problem

↓

Does the problem remind me of something I already know?

↙ Yes

Call the known algorithm
as a subroutine

No ↘

1. **Visualize** the problem with **examples**.
2. Try a **greedy** idea against your examples.
3. Identify **subproblems**, are they halves or just slightly smaller?

PollEverywhere activity

But the best way to build intuition is to practice!

- Go to pollev.com/[TA's account – update me!] and log in with your @uw.edu email.
- The activity will have a public leaderboard.
  - **Please anonymize** your screen name in the upper right corner!

tap here!