

# CSE 421 Section 10

**Final Review**

# Administrivia



# Announcements & Reminders

- HW8
  - Was Due Yesterday, Wednesday 3/8
- Final Review Session: **Saturday 3/11 @ 1:30pm-3pm in CSE2 G01**
  - We will go over the practice final, try to take it before the session if you can!
  - Hopefully we'll post a zoom recording if you can't make it!
- **Final Exam: Monday 3/13 @ 2:30-4:20 @ CSE2 G20**
  - Make sure you have it saved on your calendar!
  - If you are sick the day of the exam, let us know and we will schedule a conflict exam!

# Announcements & Reminders

Course evaluations are out! PLEASE fill out the general course evaluation for Robbie, and the section evaluation for your TAs! It helps us improve our teaching and the way the course is taught.

**This is ESPECIALLY important this quarter so we have student input on how sections worked, and we can (hopefully) improve in future quarters!!!**

# Reduction



# Problem 1 – Reduction

Consider the following problems:

HAM-PATH

Input: A **directed** graph  $G$

Output: True if there is a Hamiltonian Path in  $G$ , that is a path that visits each vertex exactly once.

HAM-CYCLE

Input: A **directed** graph  $G$

Output: True if there is a Hamiltonian Cycle in  $G$ , that is, a path  $v_0, v_1, \dots, v_n$  that visits each vertex exactly once along with the edge  $(v_n, v_0)$ .

Suppose that HAM-PATH is NP-hard. Use that fact to show HAM-CYCLE is NP-hard.

Work through this problem with the people around you, and then we'll go over it together!

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Correctness:

# Max-Flow / Min-Cut



## Problem 2 – Max-Flow / Min-Cut

A group of traders are leaving Switzerland, and need to convert their Francs (the local currency) into various international currencies. There are  $n$  traders and  $m$  currencies. Trader  $i$  has  $T_i$  Francs to convert. The bank has  $B_j$  Francs worth of currency  $j$ . Trader  $i$  is willing to trade as much as  $C_{ij}$  of his Francs for currency  $j$ .

(For example, a trader with 1000 Francs might be willing to convert up to 700 of his Francs for USD, up to 500 of his Francs for Japanese Yen, and up to 500 of his Francs for Euros).

Assuming that all traders give their requests to the bank at the same time, describe an algorithm that the bank can use to satisfy the requests (if it can).

Work through this problem with the people around you, and then we'll go over it together!

## Problem 2 – Max-Flow / Min-Cut

Assuming that all traders give their requests to the bank at the same time, describe an algorithm that the bank can use to satisfy the requests (if it can).

# DP 1: Non-Adjacent LCS



## Problem 3 – Non-Adjacent LCS

The sequence  $C = c_1, \dots, c_k$  is a non-adjacent subsequence of  $A = a_1, \dots, a_n$ , if  $C$  can be formed by selecting nonadjacent elements of  $A$ , i.e., if  $c_1 = a_{r_1}, c_2 = a_{r_2}, \dots, c_k = a_{r_k}$ , where  $r_j < r_{j+1} - 1$ . The non-adjacent Longest Common Sequence problem is given sequences  $A$  and  $B$ , find a maximum length sequence  $C$  which is a non-adjacent subsequence of both  $A$  and  $B$ .

This problem can be solved with dynamic programming. Give a recurrence that is the basis for a dynamic programming algorithm. You should also give the appropriate base cases, and explain why your recurrence is correct.

Work through this problem with the people around you, and then we'll go over it together!

## Problem 3 – Non-Adjacent LCS

Give a recurrence that is the basis for a dynamic programming algorithm. You should also give the appropriate base cases, and explain why your recurrence is correct.

# DP 2: Electoral College



# Problem 4 – Electoral College

The problem is to determine the set of states with the smallest total population that can provide the votes to win the electoral college. Formally, the problem is: Let  $p_i$  be the population of state  $i$ , and  $v_i$  the number of electoral votes for state  $i$ . All electoral votes of a state go to a single candidate, so the winning candidate is the one who receives at least  $V$  electoral votes, where  $V = \lfloor (\sum_i v_i) / 2 \rfloor + 1$ . Our goal is to find a set of states  $S$  that minimizes the value of  $\sum_{i \in S} p_i$  subject to the constraint that  $\sum_{i \in S} v_i \geq V$ .

- a) The dynamic programming solution for this problem involves computing a function OPT where  $\text{OPT}(i, v)$  gives the minimum populations of a set of states from  $1, 2, \dots, i$  such that their votes sum to exactly  $v$ . Give a recursive definition of OPT and an explanation as to why it is correct.
- b) What are the base cases for your function OPT.

Work through this problem with the people around you, and then we'll go over it together!

## Problem 4 – Electoral College

- a) The dynamic programming solution for this problem involves computing a function  $OPT$  where  $OPT(i, v)$  gives the minimum populations of a set of states from  $1, 2, \dots, i$  such that their votes sum to exactly  $v$ . Give a recursive definition of  $OPT$  and an explanation as to why it is correct.

# Problem 4 – Electoral College

b) What are the base cases for your function OPT.

# **That's All, Folks!**

**Thanks for coming to section this week!**  
**Any questions?**