

## Vertex Cover LP

Write an LP for finding the minimum weight vertex cover

A set  $S$  of vertices is a vertex cover if for every edge  $(u, v)$ ,  $u$  is in  $S$ ,  $v$  is in  $S$  or both are in  $S$ .

What are your variables, then how do you constrain them?

Let  $w(u)$  be the weight for a vertex  $u$ . You can treat  $w(u)$  as a constant.

## So, what if the graph isn't bipartite?

Big idea:  
Just round!

If  $x_u \geq \frac{1}{2}$ , round up to 1.

If  $x_u < \frac{1}{2}$ , round down to 0

[Pollev.com/robbie](https://pollev.com/robbie)

Minimize  $\sum w(u) \cdot x_u$

Subject to:

$x_u + x_v \geq 1$  for all  $(u, v) \in E$

$0 \leq x_u \leq 1$  for all  $u$ .

Two questions – is it a vertex cover? How far are we from the true minimum?

## Recall: Finding an approximation for VC

For every edge, at least one of  $u, v$  is in the minimum vertex cover.

But instead of checking which of  $u, v$  a good idea to add, just add them both!

```
While(G still has edges)
  Choose any edge (u,v)
  Add u to VC, and v to VC
  Delete u v and any edges touching them
EndWhile
```

## What have we seen this quarter?

Stable Matchings

Graph Search

BFS/DFS

Graph modeling

Greedy Algorithms

Divide and Conquer

Dynamic Programming

DP on arrays

Adding parameters

DP on trees

Network Flow

Using Max-Flow

Using Min Cuts

Assignment problems

NP-completeness

Reductions

Showing a problem is NP-hard

P vs. NP

Linear Programming

Approximation Algorithms