What's 3-SAT?

Input: A list of Boolean variables $x_1, \dots, x_n$

An expression in Conjunctive Normal Form, where each clause has exactly 3 literals.

Something like:

$$(z_i \vee z_j \vee z_k) \wedge (z_i \vee z_\ell \vee z_a) \wedge \cdots \wedge (z_a \vee z_b \vee z_c)$$

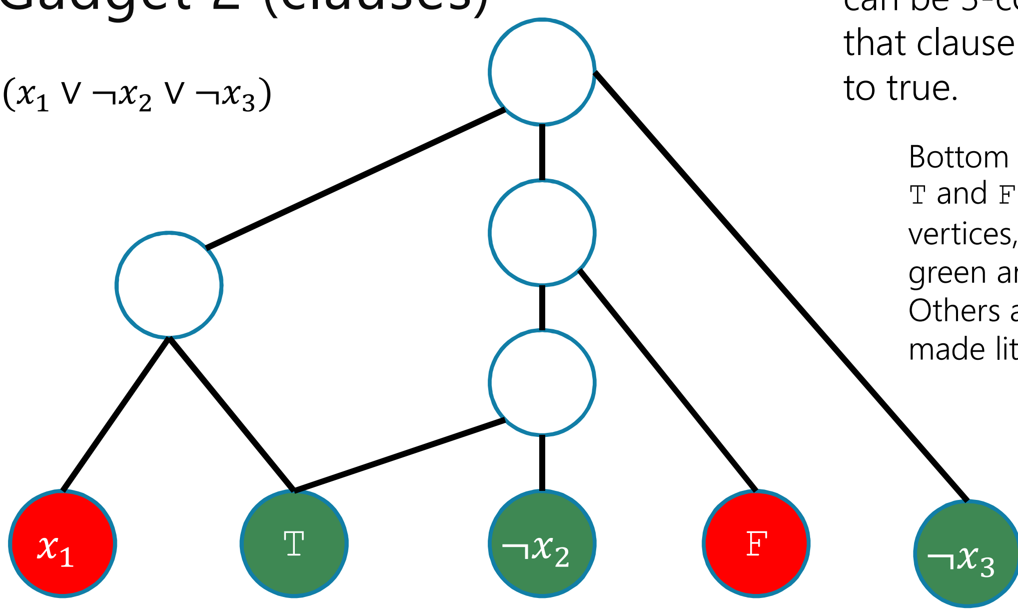
Where z is a "literal" a variable or the negation of a variable ($x_i, \neg x_j$, etc.).

Output: true if there is a setting of the variables where the expression evaluates to true, false otherwise.

Why is it called 3-SAT? 3 because you have 3 literals per clause
SAT is short for "satisfiability" can you satisfy all of the constraints?

Gadget 2 (clauses)

$(x_1 \vee \neg x_2 \vee \neg x_3)$



This tricky little graph can be 3-colored iff that clause evaluates to true.

Bottom row:
T and F are new vertices, colored green and red.
Others are already-made literal vertices

Putting it together

If the graph is 3-colorable, then there must be a satisfying assignment

I have a problem

My problem C is too difficult to solve (at least for me).

So difficult, it's probably NP-hard. How do I show it?

What does it mean to be NP-hard?

We need to be able to reduce any problem A in NP to C.

Let's choose B to be a known NP-hard problem. Since B is known to be NP-hard, $A \leq B$ for every possible A. So if we show $B \leq C$ too then $A \leq B \leq C \rightarrow A \leq C$ so every NP problem reduces to C!