Technical terms

The fancy names for the two requirements

Capacity constraints:

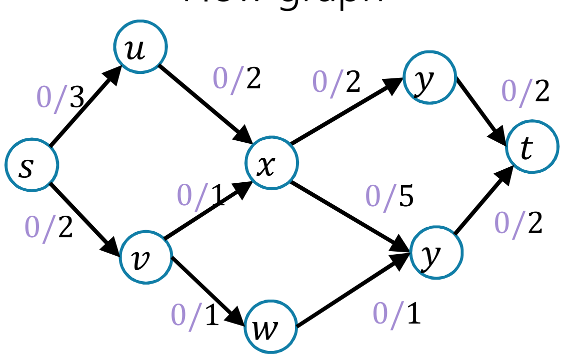
For every edge e, the flow on e is at most the capacity.

Conservation constraints:

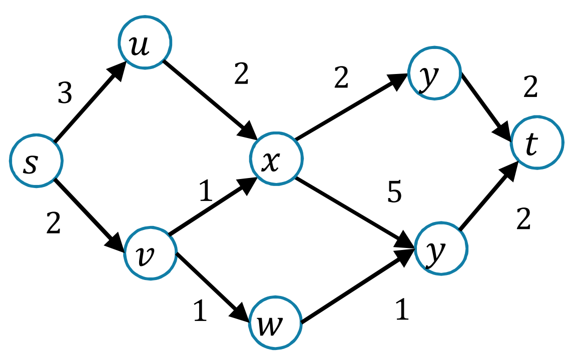
For every vertex u (except for s, t), the total flow entering u is equal to the total flow leaving u.

Example

Flow graph



Residual graph

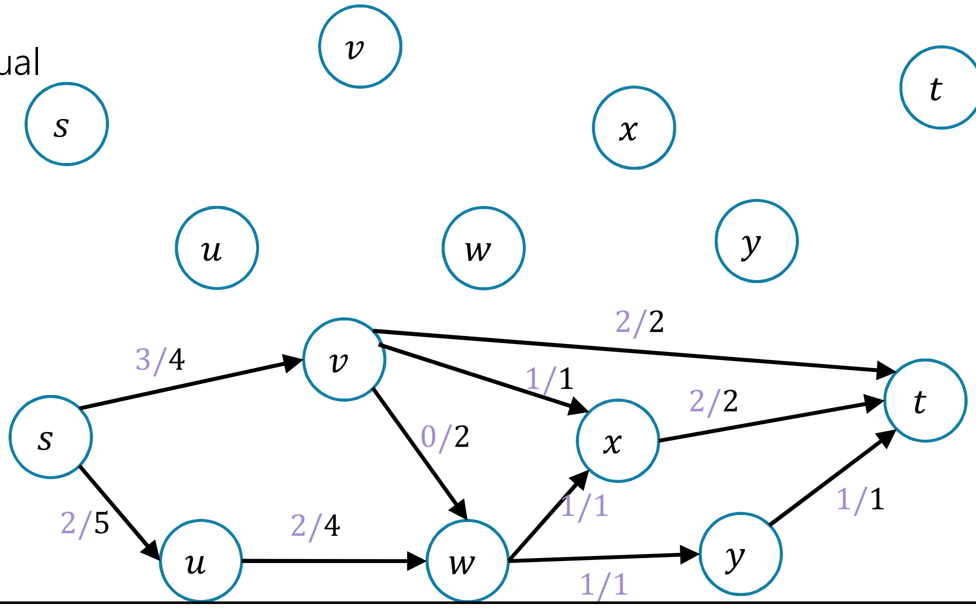


Residual starts with same edges and capacities as the flow graph.

Another Example

Residual

Flow



Step 2: Cuts limit flows ('weak duality')

Let f be any s-t flow, and (A, B) be any s-t cut.
Then $\text{val}(f) \leq \text{cap}(A, B)$

Cuts limit flows! Intuition: to get the flow to t it has to "all get through" every cut. So you can't have a flow of value more than any given cut.

Proof:

$\text{val}(f) = f^{out}(A) - f^{in}(A) \leq f^{out}(A) = \sum_{e=(u,v):u\in A, v\in B} f(e)$

$\leq \sum_{e=(u,v):u\in A, v\in B} c(e) = \text{cap}(A, B)$