Longest Increasing Subsequence

0	1	2	3	4	5	6	7
5	-6	3	6	-5	2	8	10

Longest set of (not necessarily consecutive) elements that are increasing

5 is optimal for the array above
(indices $1,2,3,6,7$; elements $-6,3,6,8,10$)

For simplicity – assume all array elements are distinct.

LIS

0	1	2	3
5	-6	3	6

$$LIS(i,j) = \begin{cases} 0 & \text{if } i < 0 \\ \mathbb{I}[A[i] \le A[j]] & \text{if } i = 0 \\ LIS(i-1,j) & \text{if } A[i] > A[j] \\ \max\{1 + LIS(i-1,i), LIS(i-1,j)\} & \text{otherwise} \end{cases}$$

j →

	0, 5	1, -6	2, 3	3, 6	4, -5	5, 2	6, 8	7, 10
0, 5	1	0	0	1	0	0	1	1
1, -6	1	1	1	1	1	1	1	1
2, 3	2	1	2	2	1	1	2	2
3, 6	2	1	2	3	1	1	3	3
4, -5	2	1	2	3	2	2	3	3
5, 2	3	1	3	3	2	3	3	3
6, 8	3	1	3	3	2	3	4	4
7, 10	3	1	3	3	2	3	4	5

i ↕

Example

What's the distance between `babyyodas` and `tastysoda?`

B	A	B		Y	Y	O	D	A	S
sub		sub	ins		sub				del
T	A	S	T	Y	S	O	D	A	

Distance: 5, one point for each colored box

Quick Checks – can you explain these?
If x has length n and y has length m, the edit distance is at most $\max(x, y)$

The distance from x to y is the same as from y to x (i.e. transforming x to y and y to x are the same)

Finding a recurrence

What information would let us simplify the problem?

What would let us "take one step" toward the solution?

"Handling" one character of x or y

i.e. choosing one of insert, delete, or substitution and increasing the "distance" by 1

OR realizing the characters are the same and matching for free.

$OPT(i, j)$ is the minimum number of insertions, deletions, and substitutions to transform $x_1 x_2 \cdots x_i$ into $y_1 y_2 \cdots y_j$. (we're indexing strings from 1, it'll make things a little prettier).