More Dynamic Programming

CSE 421 Winter 23
Lecture 11

Announcements

The midterm exam is two weeks from today. (Mon. Nov. 7)

Remember it's **in the evening**. 90 minutes, 6-7:30 PM.

We'll be in this room!

Form coming to let us know if you can't make the main exam

- Request a conflict (work/family responsibilities or something else immovable)
- Have a COVID-related reason where you know now you should not be in a big lecture hall? We'll see what we can do.
- We'll have conflicts if you're sick the night of.

Announcements

We'll have a reference sheet for you for the midterm (including, e.g., a list of some algorithms from 332 that you've been able to reference)

You'll also be able to bring your own 8.5x11 inch piece of paper with handwritten notes.

Practice materials, topics list, contents of the provided reference, etc. all coming next week.

Baby Yoda Searching

Baby Yoda has to get from the upper-right corner to the lower left.
His cradle will only let him go left and down.
He can't get past the rocks (too high) – he has to go around them (but still only going left and down)

As usual…he's hungry.
He wants to eat as many frog eggs as possible on the way.

A Recursive Function

```
FindOPT(int i,int j, bool[][] rocks, bool[][] eggs)
    if(i<0 || j < 0) return -∞
    if(rocks[i][j]) return −∞
    if(i==0 && j==0) return eggs[0][0]
    int left = FindOPT(i-1,j,rocks,eggs)
    int down = FindOPT(i,j-1,rocks,eggs)
    return Max(left,down) + eggs[i][j]
```

Recurrence Form

$$OPT(i,j) = \begin{cases} -\infty & \text{if } rocks(i,j) \text{ is true} \\ -\infty & \text{if } i < 0 \text{ or } j < 0 \\ eggs(0,0) & \text{if } i = 0 \text{ and } j = 0 \\ \max\{OPT(i-1,j), OPT(i,j-1)\} + eggs(i,j) & \text{otherwise} \end{cases}$$

Recurrences can also be used for outputs of a recursive function (not just their running times!)

This definition is a little more compact than code.

And you could write a recursive function for a recurrence like this.

Speedup

How do we go faster? Don't recalculate! **memoize**

Once you know $OPT(i, j)$ put it in an array `OPT[i][j]`

Have some initial value (null?) to mark as uninitialized

If initialized, return that.

Otherwise do the algorithm from the last slide.

How fast? Now $\Theta(rc)$.

Why? There are that many spots, each is calculated at most once and looked up at most twice.

Baby Yoda Searching

(c-1, r-1)

1	1	1	2	3	4	4	4
1	1	1	2	3	4	4	4
0	0	1	2	3	3	3	3
0	−∞	−∞	−∞	3	3	3	3
0	1	−∞	2	2	2	2	2
0	1	2	2	2	2	2	2

r-1

Y-coordinate

0

X-coordinate

0

c-1

Where's the final answer?

In the top right. Where Baby Yoda starts.

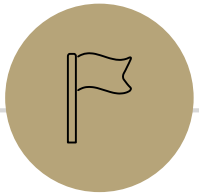
Going Bottom-up

So how does that recursion work?

What's the first entry of the table that we fill?

```
OPT[0][0]
```

Why not just start filling in there?



Robustness

Updating the Problem

A new twist on the problem.

Baby Yoda can use the force to knock over rocks.

But he can only do it once (he tires out)

How do you decide which rocks to knock over?

Could run the algorithm once for every set of rocks knocked over.

k rocks -- $\Theta(krc)$. Can we do better?

Updating the Problem

$OPT(i, j, f)$ is the maximum amount of eggs Baby Yoda can collect on a legal path from (i, j) to $(0,0)$ using the force f times to knock over rocks.

For simplicity, assume there are no rocks at the starting location (r-1,c-1)

Here was the old rule without the force – how do we update?

$$OPT(i,j) = \begin{cases} -\infty & \text{if } rocks(i,j) \text{ is true} \\ -\infty & \text{if } i < 0 \text{ or } j < 0 \\ eggs(0,0) & \text{if } i = 0 \text{ and } j = 0 \\ \max\{OPT(i-1,j), OPT(i,j-1)\} + eggs(i,j) & \text{otherwise} \end{cases}$$

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$$OPT(i, j, f) = \begin{cases} -\infty & \text{if } i < 0 \text{ or } j < 0 \text{ or } f < 0 \\ eggs(0,0) & \text{if } i = 0 \text{ and } j = 0 \text{ and } f \geq 0 \\ \max\{OPT(i-1, j, f - rocks(i-1, j)), OPT(i, j-1, f - rocks(i, j-1))\} + eggs(i, j) & \text{otherwise} \end{cases}$$

Casting Boolean as an integer (subtract 1 if you would need to knock over rocks)

Updating the Problem

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`rocks(i,j)` doesn't guarantee $-\infty$ anymore. Only if you were out of force uses before trying to jump onto that location.

Baby Yoda Searching

(c-1,r-1)

r-1

Y-coordinate

0

X-coordinate

0

c-1

What can we fill in?

a/b

a is for (x,y,0)

b is for (x,y,1)

Baby Yoda Searching

(c-1,r-1)

r-1

Y-coordinate

0

0/? 1/? 2/? 2/? 2/? 2/? 2/? 2/?

X-coordinate

0

c-1

What can we fill in?

a/b

a is for (x,y,0)

b is for (x,y,1)

Baby Yoda Searching

1/?	1/?	1/?	1/?	3/?	4/?	4/?	4/?
1/?	1/?	1/?	1/?	3/?	4/?	4/?	4/?
0/?	0/?	1/?	1/?	3/?	3/?	3/?	3/?
0/?	1/?	−∞/?	2/?	3/?	3/?	3/?	3/?
0/?	1/?	2/?	2/?	2/?	2/?	2/?	2/?
0/?	1/?	2/?	2/?	2/?	2/?	2/?	2/?

r-1

Y-coordinate

0

X-coordinate

0

c-1

What can we fill in? Everything with $f = 0$ in the same order as before.

Entries are slightly different – we're handling rocks differently.

Baby Yoda Searching

1/?	1/?	1/?	1/?	3/?	4/?	4/?	4/?
1/?	1/?	1/?	1/?	3/?	4/?	4/?	4/?
0/?	0/?	1/?	1/?	3/?	3/?	3/?	3/?
0/?	1/?	−∞/?	2/?	3/?	3/?	3/?	3/?
0/?	1/?	2/?	2/?	2/?	2/?	2/?	2/?
0/?	1/?	2/?	2/?	2/?	2/?	2/?	2/?

r-1

Y-coordinate

0

X-coordinate

0

c-1

What can we fill in?
Again from left to right, bottom to top, now filling in

Baby Yoda Searching

1/1	1/1	1/4	1/4	3/4	4/5	4/5	4/5
1/1	1/1	1/4	1/4	3/4	4/5	4/5	4/5
0/0	0/0	1/4	1/4	3/4	3/4	3/4	3/4
0/0	1/1	−∞/3	2/3	3/3	3/3	3/3	3/3
0/0	1/1	2/2	2/2	2/2	2/2	2/2	2/2
0/0	1/1	2/2	2/2	2/2	2/2	2/2	2/2

r-1

Y-coordinate

0

X-coordinate

0

c-1

What can we fill in?
Again from left to right, bottom to top, now filling in

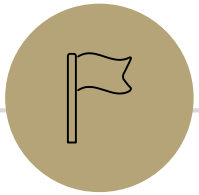
Dynamic Programming Process

1. Define the object you're looking for

2. Write a recurrence to say how to find it

3. Design a memoization structure

4. Write an iterative algorithm



Bells,Whistles, and optimiziation

Baby Yoda Searching

1/1	1/1	1/4	1/4	3/4	4/5	4/5	4/5
1/1	1/1	1/4	1/4	3/4	4/5	4/5	4/5
0/0	0/0	1/4	1/4	3/4	3/4	3/4	3/4
0/0	1/1	−∞/3	2/3	3/3	3/3	3/3	3/3
0/0	1/1	2/2	2/2	2/2	2/2	2/2	2/2
0/0	1/1	2/2	2/2	2/2	2/2	2/2	2/2

r-1

Y-coordinate

0

X-coordinate

0

c-1

So should Baby Yoda go left or down?

Which Way to Go

When you're taking the `max` in the recursive case, you can also record which option gave you the max.

That's the way to go.

We'll ask you to do that once…but for the most part we'll just have you find the number.

Optimizing

Do we need all that memory?

Let’s go back to the simple version (no using the Force)

Recurrence Form

$$OPT(i,j) = \begin{cases} -\infty & \text{if } rocks(i,j) \text{ is true} \\ -\infty & \text{if } i < 0 \text{ or } j < 0 \\ eggs(0,0) & \text{if } i = 0 \text{ and } j = 0 \\ \max\{OPT(i-1,j), OPT(i,j-1)\} + eggs(i,j) & \text{otherwise} \end{cases}$$

What values do we need to keep around?

Baby Yoda Searching

1	1	1	2	3	4	4	4
1	1	1	2	3	4	4	4
0	0	1	2	3	3	3	3
0	$-\infty$	$-\infty$	$-\infty$	3	3	3	3
0	1	$-\infty$	2	2	2	2	2
0	1	2	2	2	2	2	2

r-1

Y-coordinate

0

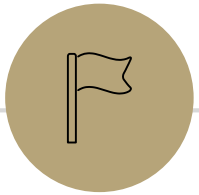
X-coordinate

0

c-1

Need one spot left and one down.

Keep one full row, and a partially full row around. $\Theta(c)$ memory.



More Problems

Maximum Contiguous Subarray Sum

We saw an $O(n \log n)$ divide and conquer algorithm.

Can we do better with DP?

Given: Array $A[]$

Output: i, j such that $A[i] + A[i+1] + \cdots + A[j]$ is maximized.

Dynamic Programming Process

1. Define the object you're looking for

2. Write a recurrence to say how to find it

3. Design a memoization structure

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Maximum Contiguous Subarray Sum

We saw an $O(n \log n)$ divide and conquer algorithm.

Can we do better with DP?

Given: Array $A[]$

Output: i, j such that $A[i] + A[i+1] + \cdots + A[j]$ is maximized.

For today: just output the value$A[i] + A[i+1] + \cdots + A[j]$.

Is it enough to know `OPT(i)`?

Trying to Recurse

0	1	2	3	4	5	6	7
5	-6	3	4	-5	2	2	4

$OPT(3)$ would give $i = 2, j = 3$

$OPT(4)$ would give $i = 2, j = 3$ too

$OPT(7)$ would give $i = 2, j = 7$ – we need to suddenly backfill with a bunch of elements that weren't optimal…

How do we make a decision on index 7? What information do we need?

What do we need for recursion?

If index i IS going to be included

We need the best subarray **that includes index $i-1$**

If we include anything to the left, we'll definitely include index $i-1$ (because of the contiguous requirement)

If index i isn't included

We need the best subarray up to $i-1$, regardless of whether $i-1$ is included.

Two Values

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Need two recursive values:

$INCLUDE(i)$: sum of the maximum sum subarray among elements from 0 to i **that includes index i** in the sum

$OPT(i)$: sum of the maximum sum subarray among elements 0 to i (that might or might not include i)

How can you calculate these values? Try to write recurrence(s), then think about memoization and running time.

Recurrences

$$INCLUDE(i) = \begin{cases} \max\{A[i], A[i] + INLCUDE(i-1)\} & \text{if } i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$OPT(i) = \begin{cases} \max\{INCLUDE(i), OPT(i-1)\} & \text{if } i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

If we include i, the subarray must be either just i or also include $i-1$.

Overall, we might or might not include i. If we don't include i, we only have access to elements $i-1$ and before. If we do, we want $INCLUDE(i)$ by definition.

Example

A

0	1	2	3	4	5	6	7
5	-6	3	4	-5	2	2	4

$OPT(i)$

0	1	2	3	4	5	6	7
5							

$INCLUDE(i)$

0	1	2	3	4	5	6	7
5							

Example

A

0	1	2	3	4	5	6	7
5	-6	3	4	-5	2	2	4

$OPT(i)$

0	1	2	3	4	5	6	7
5	5						

$INCLUDE(i)$

0	1	2	3	4	5	6	7
5	-1						

Example

A

0	1	2	3	4	5	6	7
5	-6	3	4	-5	2	2	4

$OPT(i)$

0	1	2	3	4	5	6	7
5	5	5	7	7	7	7	10

$INCLUDE(i)$

0	1	2	3	4	5	6	7
5	-1	3	7	2	4	6	10

Pseudocode

```
int maxSubarraySum(int[] A)
  int n=A.length
  int[] OPT = new int[n]
  int[] Inc = new int[n]
  inc[0]=A[0]; OPT[0] = max{A[0],0}
  for(int i=0;i<n;i++)
    inc[i]=max{A[i], A[i]+inc[i-1]}
    OPT[i]=max{inc[i], opt[i-1]}
  endFor
return OPT[n-1]
```

Recursive Thinking In General

As before, the hardest part is designing the recurrence.

It sometimes helps to think from multiple different angles.

Top-down: What's the first step to take?

Baby Yoda will first go left or down. Use recursion to find out which of left or down is better.

The farthest right operation in the string transformation will be one of insert, delete, substitute, match for free. Use recursion to find out which is best.

Recursive Thinking In General

Bottom-Up: What information could a recursive call give me that would help?

How does a path through most of the map help Baby Yoda?

Well we just need to know the values one left and one down.

The edit distance between which strings would help us compute the edit distance between our strings?

Well if we know the distance between $x_1 \dots x_{i-1}$ and $y_1 \dots y_{j-1}$ then that would tell us what happens if we substitute…that might lead you to insertions and deletions too.

Recursive Thinking In General

Some people refer to the “Optimal Substructure Property”

From the optimum (most eggs, fewest number of string operations) for a slightly smaller problem (Baby Yoda starting closer to the end, slightly smaller strings), we need to be able to build up the optimum for the full problem.

Longest Increasing Subsequence

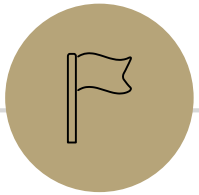
0	1	2	3	4	5	6	7
5	-6	3	6	-5	2	8	10

Longest set of (not necessarily consecutive) elements that are increasing

5 is optimal for the array above

(indices $1,2,3,6,7$; elements $-6,3,6,8,10$)

For simplicity – assume all array elements are distinct.



A Real-World Problem

Edit Distance

Given two strings x and y, we'd like to tell how close they are.

Applications?

Spelling suggestions

DNA comparison

Edit Distance

More formally:

The edit distance between two strings is:

The minimum number of **deletions, insertions,** and **substitutions** to transform string x into string y.

Deletion: removing one character

Insertion: inserting one character (at any point in the string)

Substitution: replacing one character with one other.

Example

What's the distance between `babyyodas` and `tastysoda`?

B	A	B		Y	Y	O	D	A	S
sub		sub	ins		sub				del
T	A	S	T	Y	S	O	D	A	

Distance: 5, one point for each colored box

Quick Checks – can you explain these?
If x has length n and y has length m, the edit distance is at most $\max(x, y)$

The distance from x to y is the same as from y to x (i.e. transforming x to y and y to x are the same)

Is there a greedy algorithm?

There isn't!* But maybe you think there is…

Formalize it for yourself, and see if it works on the BABYYODA, TASTYSODAS example.
If it does, try your own examples.

If it still works, ask us on Ed.

*An algorithm significantly faster than $O(n^2)$ would be huge progress in algorithm design. Please do tell us if you found one…

Finding a recurrence

What information would let us simplify the problem?

What would let us "take one step" toward the solution?

"Handling" one character of x or y

i.e. choosing one of insert, delete, or substitution and increasing the "distance" by 1

OR realizing the characters are the same and matching for free.

$OPT(i, j)$ is the edit distance of the strings $x_1 x_2 \cdots x_i$ and $y_1 y_2 \cdots y_j$. (we're indexing strings from 1, it'll make things a little prettier).

The recurrence

Poll at Pollev.com/robbie

"Handling" one character of x or y

i.e. choosing one of insert, delete, or substitution and increasing the "distance" by 1

OR realizing the characters are the same and matching for free.

Write a recurrence.

What do we need to keep track of? Where we are in each string!

Match right to left – be sure to keep track of characters remaining in each string!

The recurrence

"Handling" one character of x or y

i.e. choosing one of insert, delete, or substitution and increasing the "distance" by 1

OR realizing the characters are the same and matching for free.

What does delete look like? $OPT(i-1, j)$ (delete character from x match the rest)

Insert $OPT(i, j-1)$ Substitution: $OPT(i-1, j-1)$

Matching characters? Also $OPT(i-1, j-1)$ but only if $x_i = y_j$

The recurrence

"Handling" one character of x or y

i.e. choosing one of insert, delete, or substitution and increasing the "distance" by 1

OR realizing the characters are the same and matching for free.

Delete

Insert

Sub and matching

$$OPT(i,j) = \begin{cases} \min\{\, 1 + OPT(i-1,j),\ 1 + OPT(i,j-1),\ \mathbb{I}[x_i \neq y_j] + OPT(i-1,j-1)\} \\ j & \text{if } i = 0 \\ i & \text{if } j = 0 \end{cases}$$

"Indicator" – math for "cast bool to int"

Dynamic Programming Process

1. Define the object you're looking for

   Minimum Edit Distance between x and y

2. Write a recurrence to say how to find it

3. Design a memoization structure

4. Write an iterative algorithm

Memoization

$$OPT(i,j) = \begin{cases} \min\{\, 1 + OPT(i-1,j), 1 + OPT(i,j-1),\ \mathbb{I}[x_i \neq y_j] + OPT(i-1,j-1)\} \\ j & \text{if } i = 0 \\ i & \text{if } j = 0 \end{cases}$$

2D array n by m

`OPT[i][j]` is $OPT(i,j)$

Edit Distance

$\mathbf{OPT}(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0										
T 1										
A 2										
S 3										
T 4										
Y 5										
S 6										
O 7										
D 8										
A 9										

Edit Distance

B	A	B	Y
T	A	S	

Current subproblem: edit dist between BABY and TAS

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1									
A 2	2									
S 3	3									
T 4	4									
Y 5	5									
S 6	6									
O 7	7									
D 8	8									
A 9	9									

Edit Distance

B	A	B	Y
T	A	S	

Delete: get rid of Y (cost 1)
BAB
TAS

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1									
A 2	2									
S 3	3									
T 4	4									
Y 5	5									
S 6	6									
O 7	7									
D 8	8									
A 9	9									

Edit Distance

B	A	B	Y
T	A	S	

Insert: insert S (cost 1)
BABY
TA

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1									
A 2	2									
S 3	3									
T 4	4									
Y 5	5									
S 6	6									
O 7	7									
D 8	8									
A 9	9									

Edit Distance

B	A	B	Y
T	A	S	

Sub: Transform Y to S (cost 1)
BAB
TA

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1									
A 2	2									
S 3	3									
T 4	4									
Y 5	5									
S 6	6									
O 7	7									
D 8	8									
A 9	9									

Edit Distance

B	A	B	Y
T	A	S	

Gold entry will be min of:

1 + delete

1 + insert

1 + sub

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1									
A 2	2									
S 3	3									
T 4	4									
Y 5	5									
S 6	6									
O 7	7									
D 8	8									
A 9	9									

Edit Distance

$1 + 2$
$1 + 3$
$1 + 2$
Min: 3

Gold entry will be min of:
$1 +$ delete
$1 +$ insert
$1 +$ sub

OPT(i,j)	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	3	3	3	4	5	6	7	8
Y 5	5	5	4	4	3					
S 6										
O 7										
D 8										
A 9										

Edit Distance

Fill in the next two entries. Be careful with the sub/match distinction!

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	3	3	3	4	5	6	7	8
Y 5	5	5	4	4	3					
S 6										
O 7										
D 8										
A 9										

Edit Distance

Fill in the next two entries. Be careful with the sub/match distinction!

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	3	3	3	4	5	6	7	8
Y 5	5	5	4	4	3	**3**	**4**			
S 6										
O 7										
D 8										
A 9										

Y's match, so sub is free!

Edit Distance

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	3	3	3	4	5	6	7	8
Y 5	5	5	4	4	3	3	4	5	6	7
S 6	6	6	5	5	4	4	4	5	6	6
O 7	7	7	6	6	5	5	4	5	6	7
D 8	8	8	7	7	6	6	5	4	5	6
A 9	9	9	8	8	7	7	6	6	4	5

Edit Distance

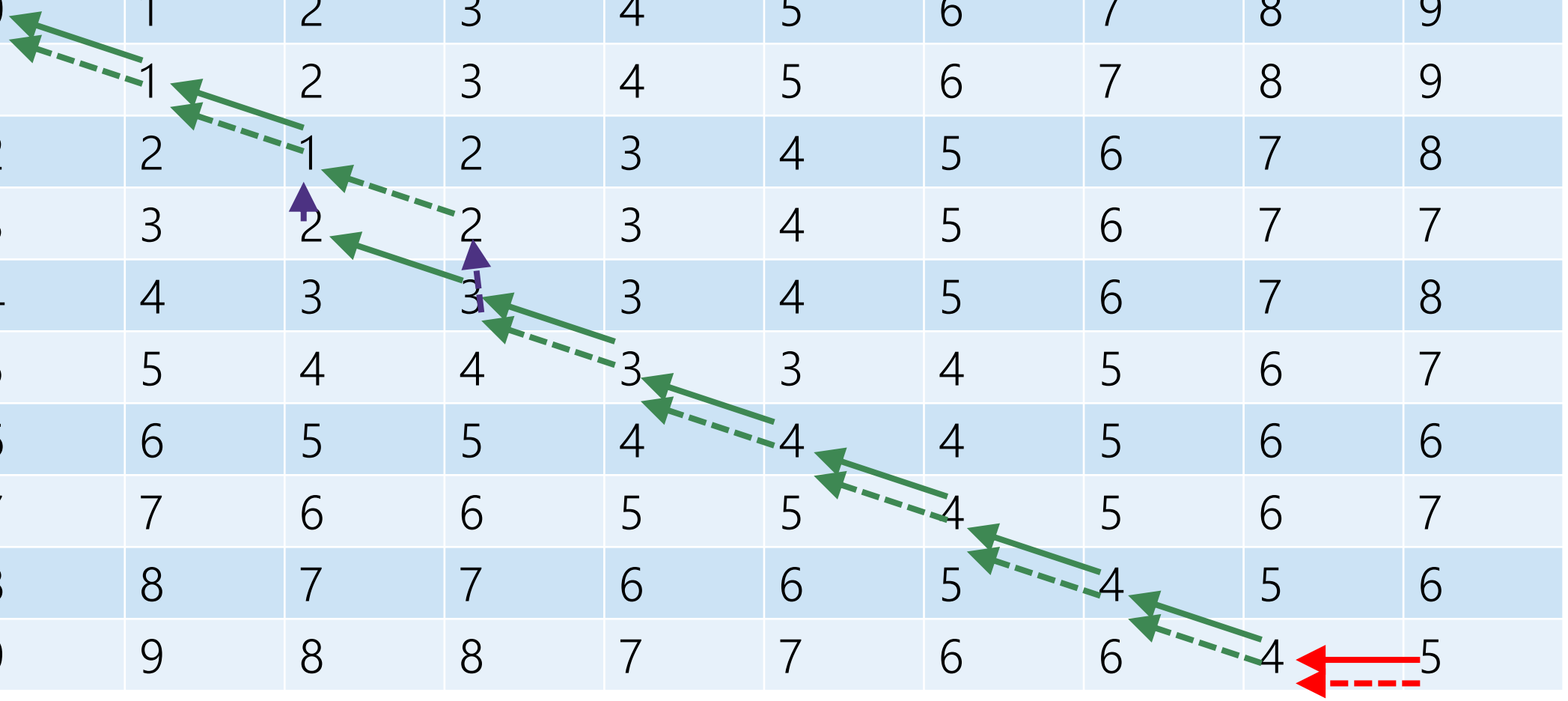
Can recover the choices, just like with Baby Yoda.

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	3	3	3	4	5	6	7	8
Y 5	5	5	4	4	3	3	4	5	6	7
S 6	6	6	5	5	4	4	4	5	6	6
O 7	7	7	6	6	5	5	4	5	6	7
D 8	8	8	7	7	6	6	5	4	5	6
A 9	9	9	8	8	7	7	6	6	4	5

Edit Distance

When there are ties, there may be more than one set of choices.

$OPT(i,j)$	0	B, 1	A, 2	B, 3	Y, 4	Y, 5	O, 6	D, 7	A, 8	S, 9
0	0	1	2	3	4	5	6	7	8	9
T 1	1	1	2	3	4	5	6	7	8	9
A 2	2	2	1	2	3	4	5	6	7	8
S 3	3	3	2	2	3	4	5	6	7	7
T 4	4	4	3	3	3	4	5	6	7	8
Y 5	5	5	4	4	3	3	4	5	6	7
S 6	6	6	5	5	4	4	4	5	6	6
O 7	7	7	6	6	5	5	4	5	6	7
D 8	8	8	7	7	6	6	5	4	5	6
A 9	9	9	8	8	7	7	6	6	4	5



Dynamic Programming Process

1. Define the object you're looking for

   Minimum Edit Distance between x and y

2. Write a recurrence to say how to find it

   ✅

3. Design a memoization structure

   $m \times n$ Array

4. Write an iterative algorithm

   Outer loop: increasing rows (starting from 1)
   Inner loop: increasing column (starting from 1)