

# Reference Sheet

Unless explicitly stated otherwise, you may use any algorithm discussed in this class or 332 to solve a problem. In particular, you may use any of these functions as libraries (this list is not exhaustive).

## Graph Search

- `TwoColor( $G$ )` returns `True` if  $G$  can be 2-colored (i.e., is bipartite), `False` otherwise. Running time  $\Theta(m + n)$
- `ConnectedComponents( $G$ )` finds the connected components of an undirected graph  $G$ . You may assume you get any reasonable representation of this information. Running time  $\Theta(m + n)$
- `StronglyConnectedComponents( $G$ )` finds the strongly connected components of a directed graph  $G$ . You may assume you get any reasonable representation of the information. Running time  $\Theta(m + n)$
- `TopologicalSort( $G$ )` returns a list of vertices of a directed graph  $G$  in topological order, or `null` if the graph has a cycle. Running time  $\Theta(m + n)$
- `CondensationGraph( $G$ )` returns the condensation of a directed graph  $G$  (a.k.a., the “meta-graph” of  $G$  or “graph of SCCs of  $G$ ”). Running time  $\Theta(m + n)$
- `Dijkstra( $G, s, t$ )` finds the length of the shortest path from  $s$  to  $t$  in a (non-negative) weighted, directed graph  $G$ . If  $t$  is `null`, the distances from  $s$  are stored in every vertex. Running time  $\Theta(m + n \log n)$
- `Prims( $G$ )` finds the minimum spanning tree of a (weighted, undirected) graph  $G$ . Running time  $\Theta(m \log n)$

## Arrays

- `QuickSelect( $A, k$ )` returns the value which would be at index  $k$  of  $A$  if  $A$  were sorted. Running time  $\Theta(n)$
- `MaxSubarraySum( $A$ )` returns the sum of the maximum sum (contiguous) subarray of  $A$ . Running time  $\Theta(n)$
- `MergeSort( $A$ )` returns the sorted version of  $A$ . Running time  $\Theta(n \log n)$

## Others

- `GaleShapley(riderPrefs, horsePrefs)` returns the rider-optimal stable matching. Running time  $\Theta(n^2)$  for  $n$  riders and  $n$  horses
- `2dClosestPoints( $A$ )` returns the distance between the two closest points of  $A$  (where  $A$  contains vectors in  $\mathbb{R}^2$ ). Running time  $\Theta(n \log n)$
- `EditDistance( $x, y$ )` returns the edit distance between strings  $x$  and  $y$ . Running time  $\Theta(mn)$  for strings of length  $m, n$

*There's more information on the back!*

## Other information

**Master Theorem** For a recurrence of the following form, where  $a, b, c, d$  are constants

$$T(n) = \begin{cases} d & \text{if } n \text{ is at most some constant} \\ aT\left(\frac{n}{b}\right) + f(n) & \text{otherwise} \end{cases}$$

Where  $f(n)$  is  $\Theta\left(n^c \cdot \log^k(n)\right)$  for  $k \geq 0, a \in \mathbb{Z}^+, c \geq 1$

- If  $\log_b(a) < c$  then  $T(n) \in \Theta\left(n^c \cdot \log^k(n)\right)$
- If  $\log_b(a) = c$  then  $T(n) \in \Theta\left(n^c \cdot \log^{k+1}(n)\right)$
- If  $\log_b(a) > c$  then  $T(n) \in \Theta\left(n^{\log_b(a)}\right)$