CSE 421 Lecture 6

1 Minimum Lateness

Theorem 1. *For the minimum lateness problem, greedy algorithm (ordered by deadline) is optimal.*

The proof involves the following operations:

Definition 2. We call a pair of jobs (i, j) is valid if

- $d_i \leq d_j$
- Job i is scheduled immediately after job j.

Lemma 3. *Swapping a valid pair of jobs does not increase the maximum lateness.*

Proof:
Let S be the schedule before swap and let S' be the schedule after.
Let i, j be the pair to swap.
1) All job except i and j has the same lateness because the finish time is unchanged.
2) The lateness of job i, we have

$$L'_i < L_i.$$

3) The lateness of job j,

$$\begin{aligned} L'_j &= f'_j - d_j \\ &= f_i - d_j \\ &\leq f_i - d_i \\ &= L_i \end{aligned}$$

Combining all cases, we have $\max L'_k \leq \max L_k$.

Now, we go back to our main proof:
Proof of the main theorem:
Let A be the schedule given by greedy method.
Let O be some optimal schedule.
Using bubble sort, we can transform the schedule O to schedule A using valid swap, namely, we have

$$O = O_1 \rightarrow O_2 \rightarrow \cdots \rightarrow O_T = A$$

such that $O_{t+1}$is obtained from swapping a valid pair from O_t.
By previous lemma, we have

$$L(O) \geq L(O_1) \geq L(O_2) \geq \cdots \geq L(A).$$

Since O is optimal, A is optimal.