1 Interval Scheduling

Theorem 1. For the interval scheduling problem, greedy methods (ordered by finish time) gives an optimal solution.

Proof:
Let $i_1, i_2, i_3, \ldots$ be the $k$ jobs picked by greedy method (order by finish time).
Let $j_1, j_2, j_3, \ldots$ be any valid solution with the $m$ jobs (order by finish time).
We show that “$k \geq r$ and $f(i_r) \leq f(j_r)$” by induction on $r \in \{1, 2, \ldots, m\}$.

Base case: $r = 1$:
Greedy picks the job according to min finish time. So, $k \geq 1$ and $f(i_1) \leq f(j_1)$.

Induction:
By hypothesis, $f(i_{r-1}) \leq f(j_{r-1})$.
Since $j_{r-1}, j_r$ does not overlap, $f(j_{r-1}) \leq s(j_r)$.
So, job $j_r$ is one of the candidate greedy method looks at.
Since greedy picks the job finishes earliest among the candidates, $k \geq j$ and $f(i_r) \leq f(j_r)$.

2 Interval Partitioning

Theorem 2. For the interval partitioning problem, greedy methods (ordered by start time) gives an optimal solution.

Proof:
Let $d$ is the number of classroom greedy method uses.
Classroom $d$ is opened because some lecture $j$ is not compatible with another $d - 1$ lectures. Look at that time, there are $d$ lectures at the same time.
So, the depth is $\geq d$.
So, any solution requires at least $d$ classrooms.