Example 2: Vertex Cover $\leq^1_p$ Indep Set

Reduction.

- Given an instance of vertex cover $G = (V, E)$ with the integer $k$.
- If there is a $G$ has a independent set of size at least $n - k$
  - Return YES ($G$ has a vertex cover of size at most $k$.)
- Else
  - Return NO.

Runtime. This reduction only make one call to independent set and polynomial time extra work.

Correctness.

Claim. For any graph $G = (V, E)$, $S$ is an independent set if and only if $V - S$ is a vertex cover.

Proof. Only if part:

Let $S$ be an independent set of $G$. Then, $S$ has at most one endpoint of every edge of $G$. Hence, $V - S$ has at least one endpoint of every edge of $G$. Therefore, $V - S$ is a vertex cover.

If part:

Let $V - S$ be a vertex cover. Then, there is no edge between vertices of $S$. (otherwise, $V - S$ is not a vertex cover). So, $S$ is an independent set.

In particular, the claim shows that $G$ has a independent set of size $\geq n - k$ if and only if $G$ has a vertex cover of size $\leq k$. This proves the correctness of the algorithm.