Example 2: Vertex Cover $\leq_p^1$ Indep Set

Reduction.

- Given an instance of vertex cover $G = (V, E)$ with the integer k.
- If there is a G has a independent set of size at least $n - k$
  - Return YES (G has a vertex cover of size at most k.)
- Else
  - Return NO.

Runtime. This reduction only make one call to independent set and polynomial time extra work.

Correctness.

Claim. For any graph $G = (V, E)$, S is an independent set if and only if $V - S$ is a vertex cover.

Proof. Only if part:

Let S be an independent set of G. Then, S has at most one endpoint of every edge of G. Hence, $V - S$ has at least one endpoint of every edge of G. Therefore, $V - S$ is a vertex cover.

If part:

Let $V - S$ be a vertex cover. Then, there is no edge between vertices of S. (otherwise, $V - S$ is not a vertex cover). So, S is an independent set. □

In particular, the claim shows that G has a independent set of size $\geq n - k$ if and only if G has a vertex cover of size $\leq k$. This proves the correctness of the algorithm.