

CSE 421 LECTURE 1

Theorem 1. *G-S algorithm outputs a stable matching in $O(n^2)$ steps.*

Proof. From the algorithm, we note that

- (Obs 1) man proposed in decreasing order of preference.
- (Obs 2) woman's partner get better and better. Never unmatched once matched.

Termination/Runtime:

- Each step, there is some woman w that the free man m never proposed to.
 - Supposed on contrary that every woman has been proposed by m .
 - (Obs 2) shows every woman is matched.
 - So, every man is matched also (since it is 1 to 1 matching).
 - This contradicts to the fact m is a free man.
- Runtime
 - Let S be the set of proposed (m, w) pair.
 - Each step, some man m proposed to some woman w (that m never proposed to).
 - Hence, the size of S is increased by 1.
 - Since $|S| \leq n^2$, there are at most n^2 steps.

Correctness:

- Perfectness
 - Terminate implies there is no free man
 - Hence, every man and woman are matched.
- Stability
 - Consider any unmatched pair (m, w)
 - Case 1: m never proposed to w .
 - * (Obs 1) shows m prefer his current partner than w .
 - Case 2: m proposed to w .
 - * (Obs 2) shows w prefer her current partner than m
 - In both cases, (m, w) is stable.

□