CSE 421

Midterm review

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Homework 3 Comments

Keep pseudocode concise and clear
• Avoid writing out the sorting algorithm (or any algorithm we taught).
  – This is difficult to read and may lead to more errors.
  – Better: Sort jobs by \( t_i/w_i \) in ascending order.
• Avoid long notation (such as jobs[i].w)
  – Better: \( w_i \) (Defined in the problem. So, you can just use it)

Proof
• Bad: My algorithm does X at each step, which brings me closer to a correct solution, therefore my algorithm is correct.
• Advice: Split the proof into multiple steps.
  – Example: To prove the algorithm find a certain subset \( X^* \),
  – Split it to \( X \subset X^* \) and \( X^* \subset X \) where X is the output of the algorithm.
  – Example: To prove the algorithm compute something
  – Use induction and prove the \( k \)-step of the algorithm computes (something)
Format

Coverage: Lecture 1 – 12
Time: 50 min

Format:
18% true / false
27% fill in the blank
55% long question
• 1 question on graphs
• 1 question on greedy methods
• 1 question on divide and conquer

Only 1 question requires proof.
(Look at the sample midterms in the website)
Exercise for Midterm

Given an undirected graph $G$ with $n$ vertices and $m$ edges. Each edge represents a highway or a flight. Let $c_e$ be the # hours needed to cross edge $e$.

Suppose that
- it takes 3 extra hours to pass through the security in airport.
- No extra hour for transferring from one flight to another.

Give a polynomial time algorithm to find the fastest way to go from vertex $s$ to vertex $t$.

(Hints: Reduction.)
Exercise for Midterm

Given an undirected graph $G$ with $n$ vertices and $m$ edges. Each edge represents a highway or a flight. Let $c_e$ be the \# hours needed to cross edge $e$.

Suppose that

- it takes 3 extra hours to pass through the security in airport.
- No extra hour for transferring from one flight to another.
- You cannot take more than 3 flights in the whole trip.

Give a polynomial time algorithm to find the fastest way to go from vertex $s$ to vertex $t$.
(Hints: Reduction.)
Exercise for Midterm

Given a sequence of increasing integer $a_1, a_2, \ldots, a_n$. Assume there is $i$ such that $a_i = i$. Give an algorithm to find such $i$ in $O(\log n)$ time.

(Hints: Reduction.)
Exercise for Midterm

Given a complete binary tree with root $r$ and $n$ vertices. Give an algorithm to find $k$ leaves of the tree in $O(k + \log n)$ time.

(Note: Every non-leaf of a complete binary tree has two children.)
Exercise for Midterm

Given a weighted directed acyclic graph with $n$ vertices and $m$ edges. Give an $O(n + m)$ time algorithm to find the shortest path distance from vertex $s$ to all other vertices. (Hints: Topological sort.)
Exercise for Midterm

Given a connected graph with $n$ vertices and $m$ edges with $m \geq n$. Give an $O(n)$ time algorithm to find a cycle. 
(Hints: Tree.)