CSE421: Introduction to Algorithms January 11, 2022

Homework 1

Yin Tat Lee Due: Jan 12, 2022 (before class)

Please see Edstem for general guidelines about homework problems.

1. (8 Marks) In the lecture, we showed that the G-S algorithm takes $O(n^2)$ steps, but we did not discuss the total runtime. Write pseudocode to describe how each step of the algorithm can be implemented and discuss its runtime. (A faster algorithm gets more credit.)

2. (4 Marks) Arrange the following in increasing order of asymptotic growth rate. No proof is required.

   (a) $f_1(n) = n^5/10$

   (b) $f_2(n) = 2^{(\log n)^{2/3}}$

   (c) $f_3(n) = (\log n)^{\sqrt{\log n}}$

   (d) $f_4(n) = 5n$

   (e) $f_5(n) = 2^{\log_3 n}$

   (f) $f_6(n) = \exp(n)$

3. (8 Marks) Prove that in any tree with n vertices, the number of vertices with degree ≥ 6 is at most $(n-1)/3$.

4. (10 Marks) YinTat Shipping Lines, Inc., is a shipping company that owns n ships and provides service to n ports with $n \leq 28$. Each of its ships has a schedule that says, for each day of the month, which of the ports it's currently visiting, or whether it's out at sea. Each ship visits each port for exactly one day during the month. For safety reasons,

   (*) No two ships can be in the same port on the same day.

   The company wants to perform maintenance on all the ships this month, via the following scheme. They want to truncate each ship's schedule: for each ship S_i , there will be some day when it arrives in its scheduled port and simply remains there for the rest of the month (for maintenance). This means that S_i will not visit the remaining ports on its schedule (if any) that month. So the truncation of S_i's schedule will simply consist of its original schedule up to a certain specified day on which it is in a port P; the remainder of the truncated schedule simply has it remain in port P.

   The question is the following: Given the schedule for each ship, find a truncation of each so that condition (*) continues to hold: no two ships are ever in the same port on the same day. Show that such a set of truncations can always be found, and give an algorithm to find them.

   Hints: If you want to use the G-S algorithm as a black-box, you only need to specify what "man" and "woman" correspond to, what their preferences are, and why the resulting stable matching corresponds to a set of truncations required in the problem.

5. (**Extra Credit)** We call an algorithm a truthful mechanism if no one ends up better off by lying about his or her preferences.

   In this question, we study whether the G-S algorithm is truthful. Prove one of the following two things:

   (a) Give a proof that, for any set of preference lists, for any man/woman, he/she cannot improve his/her partner by lying about his/her preferences; or

   (b) Give an example of a set of preference lists for which a man/woman can improve his/her partner by lying about his/her preferences.