CSE421: Design and Analysis of Algorithms	April 19, 2022
Shayan Oveis Gharan	Problem Solving Session 3

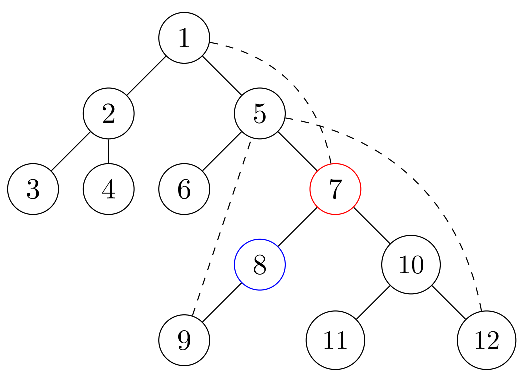
P1) The main goal of this question is to answer the problem in part (d). If you prefer you can ignore parts (a,b,c) and solve (d) directly.

a) **Optional** Let T be a tree with n vertices. Prove that there is a **unique** path in T between each pair of vertices.

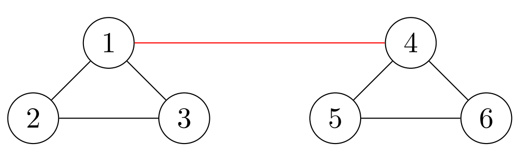
b) **Optional** Let $G = (V, E)$ be a connected undirected graph and $T \subseteq E$ be a spanning tree of G. Prove that for any edge $e \in T$, $G - e$ is connected iff there is an edge $f = (u, v) \in E - T$ such that the unique path between u, v in T has the edge e.

c) Show how to modify the code for recursive depth-first search of undirected graphs to obtain an $O(n+m)$ time algorithm that (i) assigns each node v a number, dfsnum(v), indicating a sequence number for when v was first visited by DFS, and computes $min(v)$ for each node v, the smallest dfsnum of any node that was encountered in the recursive call $dfs(v)$.

For example, in the following picture edges of the DFS tree are marked in solid (and non-tree edges in dashed). Every node is labelled with its dfsnum. So, $min(.)$ for the red node is 1 and min of the blue node is 5.



d) Given a graph $G = (V, E)$ with n vertices and m edges, design an $O(m+n)$ time algorithm that for any edge $e \in E$ outputs if $G - e$ is connected. For example, given the following graph you should output "yes" for all black edges and "no" for the red edge.



P2) Prove or disprove: Every directed graph with n vertices and at least $n(n-1)/2 + 1$ directed edges has a cycle.