Loose End 2: Input Size

The definitions of both P and NP refer to the "size" of the input.

P (stands for "Polynomial")

The set of all decision problems that have an algorithm that runs in time $O(n^k)$ for some constant k (on input of size n).

NP (stands for "nondeterministic polynomial")

The set of all decision problems such that for every YES-instance (of size n), there is a certificate (of size $O(n^k)$) for that instance which can be verified in polynomial time.

What does "size" mean?

Approximation Ratio

For a minimization problem (find the shortest/smallest/least/etc.)

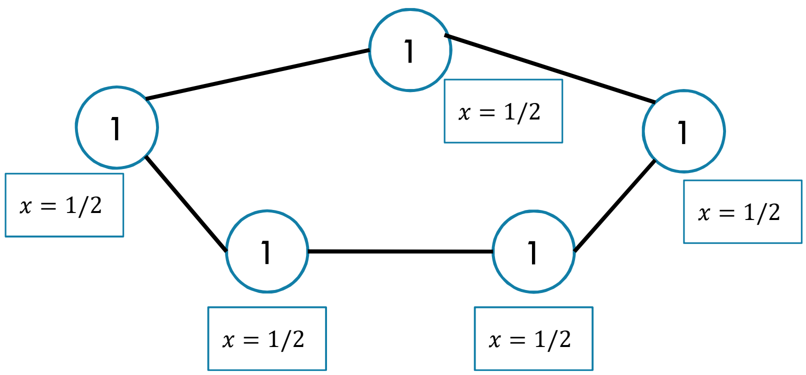
If $OPT(\mathrm{G})$ is the value of the best solution for G, and $ALG(G)$ is the value that your algorithm finds, then ALG is an α approximation algorithm if for every G,

$$\alpha \cdot OPT(G) \geq ALG(G)$$

i.e. you're within an α factor of the real best.

Non-Bipartite

What if our original graph isn't bipartite?



The LP finds a fractional vertex cover of weight 2.5

There's no "real"/integral VC of weight 2.5. – lightest is weight 3.

There's a "gap" between integral and fractional solutions.

So, what if the graph isn't bipartite?

Big idea:

Just round!

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If $x_u \geq \frac{1}{2}$, round up to 1.

If $x_u < \frac{1}{2}$, round down to 0

Minimize $\sum w(u) \cdot x_u$
Subject to:
$x_u + x_v \geq 1$ **for all** $(u, v) \in E$
$0 \leq x_u \leq 1$ **for all** u.

Two questions – is it a vertex cover? How far are we from the true minimum?