Let's Do A Reduction

4 steps for reducing (decision problem) A to problem B.

1. Describe the reduction itself (i.e. the algorithm, with call(s) to a library for problem B)
2. Make sure the running time would be polynomial (in lecture, we'll sometimes skip writing out this step).
3. Argue that if the correct answer (to the instance for A) is YES, then our algorithm answers YES.
4. Argue that if the correct answer (to the instance for A) is NO, then our algorithm answers NO.

Correctness?

```
2ColorCheck(Graph G)
     Let H be a copy of G
     Add a vertex to H, attach it to all
       other vertices.
     Bool answer = 3ColorCheck(H)
     return answer
```

TWO statements to prove: ("two directions")

If the correct answer for G is YES, then we say YES

If the correct answer for G is NO, then we say NO

Some New Problems

Here are some new problems. Are they in NP?

If they're in NP, what is the "certificate" when the answer is yes?

COMPOSITE – given an integer n is it composite (i.e. not prime)?

MAX-FLOW – find a maximum flow in a graph.

VERTEX-COVER – given a graph G and an integer k, does G have a vertex cover of size at most k?

NON-3-Color – given a graph G, is it not 3-colorable?

P (stands for "Polynomial")

The set of all decision problems that have an algorithm that runs in time $O(n^k)$ for some constant k (on input of size n).

NP (stands for "nondeterministic polynomial")

The set of all decision problems such that for every YES-instance (of size n), there is a certificate (of size $O(n^k)$) for that instance which can be verified in polynomial time.

NP-hard

The problem B is NP-hard if
for all problems A in NP, A reduces to B.

NP-Complete

The problem B is NP-complete if B is in NP
and B is NP-hard