Approximation Ratio

For a minimization problem (find the shortest/smallest/least/etc.)

If $OPT(\mathrm{I})$ is the value of the best solution for input I, and $ALG(I)$ is the value that your algorithm finds, then ALG is an α approximation algorithm if for every I,

$$\alpha \cdot OPT(G) \geq ALG(G)$$

i.e. you're always within an α factor of the real best.

Usually use big-$\mathcal{O}$ notation on the ratio.

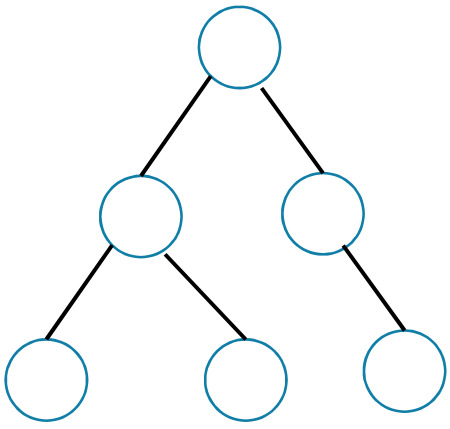
Vertex Cover

Vertex Cover

A set S of vertices is a vertex cover if for every edge (u, v): u is in S, or v is in S, (or both)

Find the minimum vertex cover in a graph.

We're picking a set of ***vertices*** so that the ***vertices*** cover every edge.



Finding an approximation for Vertex Cover

Here's Idea 2

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While(G still has edges)
     Choose any edge (u,v)
     Add u to VC, and v to VC
     Delete u v and any edges touching them
EndWhile
```

Why? At least one of u, v is in the vertex cover. We know we're not getting the exact optimal, so…don't try. At least one of the two was a good decision.

Run the greedy algorithm

Let $U = \{1,2,3,4,5,6,7,8\}$

$S = \{\ \{1,2,3\}, \{2,4\}, \{6,8\}, \{3,5,7\}, \{5,7,8\}, \{2,5,6\}, \{4\}\ \}$