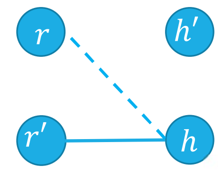
Proposer-Optimality

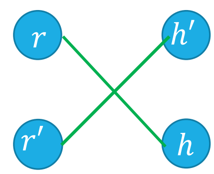
Every member of the proposing side is matched to the favorite of their feasible partners.



Intuition:

The riders start at the top of their lists. For the claim to be false, some rider r has to be the first to be rejected by their favorite feasible horse, h.

When that happens, h says it prefers some r' (and it does that while r' is still in the "favorite feasible partner" or "too good for you" sections of their list). So r' and h would block any matching

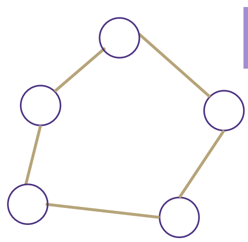


A detailed application

Bipartite (also called "2-colorable")

A graph is bipartite (also called 2-colorable) if the vertex set can be divided into two sets V_1, V_2 such that the only edges go between V_1 and V_2.

Called "2-colorable" because you can "color" V_1 red and V_2 blue, and no edge connects vertices of the same color.



If a graph contains an odd cycle, then it is not bipartite.

Try the example on the right, then proving the general theorem in the light purple box.

Help Robbie figure out how long to make the explanation
Pollev.com/robbie

BFS With Layers

```
search(graph)
   toVisit.enqueue(first vertex)
   mark first vertex as seen
   toVisit.enqueue(end-of-layer-marker)
   l=1
   while(toVisit is not empty)
      current = toVisit.dequeue()
      if(current == end-of-layer-marker)
          l++
          toVisit.enqueue(end-of-layer-marker)
      current.layer = l
      for (v : current.neighbors())
         if (v is not seen)
            mark v as seen
            toVisit.enqueue(v)
```

It's just BFS!

With some extra bells and whistles.

Lemma 3

If a graph has no odd-length cycles, then it is bipartite.