Gale-Shapley Algorithm

```
Initially all r in R and h in H are free
while there is a free r
     Let h be highest on rs list that r has not proposed to
     if h is free
         match (r,h)
     else //h is not free
         Let r′ be the current match of h.
         if h prefers r to r′
               unmatch (r′,h)
               match (r,h)
```

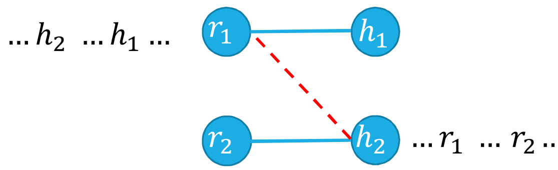
Claim 1: If r proposed to the last horse on their list, then all the horses are matched.

Try to prove this claim, i.e. clearly explain why it is true. You might want some of these observations:

- Observation A: **r's proposals get worse (for r).**
- Observation B: **Once h is matched, h never becomes free again.**
- Observation C: **h's partners cannot get worse (for h).**

Hint: r must have been rejected a lot – what does that mean?

Claim 4: The matching has no blocking pairs.



How did r_1 end up matched to h_1?

Multiple Stable Matchings

Suppose we take our algorithm and let the horses do the "proposing" instead.