NAME: __________________________

CSE 421
Introduction to Algorithms
Midterm Exam Spring 2020

DIRECTIONS:

• Answer the problems on the exam paper.

• Justify all answers with proofs (except for Problem 1),
  unless the facts you need have been stated or proven in
  class, or in Homework, or in sample-midterm.

• If you need extra space use the back of a page or two
  additional pages at the end

• You have 70 minutes to complete the exam.

• Please do not turn the exam over until you are in-
  structed to do so.

• Good Luck!
1. (25 points, 5 each) For each of the following problems answer True or False (no proof needed).

(a) $2^n = O(2^n)$.

(b) Every acyclic graph with $n$ vertices has at most $n - 1$ edges.

(c) Let $G$ be a weighted connected graph such the costs of all edges are distinct, i.e., $c_e \neq c_f$ for all $e, f$, and let $T$ be the minimum spanning tree of $G$. If we increase the cost of every edge by 1 then $T$ is still the minimum spanning tree.

(d) Let $G$ be a weighted connected graph, and let $P$ be the shortest path from $s$ to $t$. If we increase the cost of every edge by 1, then $P$ is still the shortest path from $s$ to $t$.

(e) For any weighted connected undirected graph $G$, Dijkstra’s algorithm finds the shortest path from $s$ to all vertices of $G$.

(f) If $T(n) \leq 16T(n/4) + n^2$, $T(1) = 1$, then $T(n) = O(n^2)$. 
2. Design a polynomial time algorithm that given a connected (undirected) graph $G$ with $n$ vertices and $m$ edges and an integer $1 \leq k \leq n - 2$, outputs $k$ vertices of $G$ such that if we remove all of these vertices the remaining graph is still connected.

**Extra credit (5 points)** You receive extra credit if your algorithm runs in $O(m + n)$.

For example, given the following graph and $k = 2$, you can remove vertices $c, d$ such that the remaining graph (which has only $a, b$ vertices) is connected.

![Diagram of a graph with vertices a, b, c, d and edges connecting them in a way that removing c and d makes the graph connected.](image-url)
3. Let $G$ be an undirected graph (not necessarily connected) with $n$ vertices and $m$ edges. Design a polynomial time algorithm that outputs if $G$ can be oriented such that the indegree of every vertex is at least 1. Output “yes” if such an orientation exists and “no” otherwise.

For example, you should output “no” for the left graph and “yes” for the right graph.

![Diagram](https://via.placeholder.com/150)
4. Given a tree $T$ with $n$ vertices such that the degree of every vertex is at most 10. We say $T$ is nicely colored if we can color vertices of $T$ such that for any vertex $v$, neighbors of $v$ have distinct colors. Prove $T$ can be nicely colored with at most 10 colors.

For example, the right picture is a nice coloring of the tree in the left and uses at most 10 colors. For instance, observe that neighbors 0, 3, 4 of vertex 2 have distinct colors red, blue and green. Note that the color of a vertex $v$ does not have to be different from its neighbors.