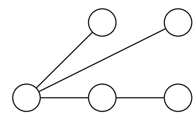
CSE421: Design and Analysis of Algorithms	April 7, 2020

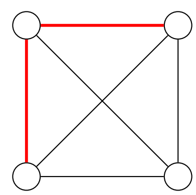
Homework 2

Shayan Oveis Gharan Due: April 15, 2021 at 11:59 PM

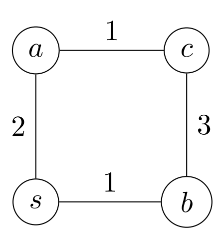
P1) (20 points) Let G be a tree. Use induction to prove that the number of leaves of G is at least the number of vertices of degree at least 3 in G. For example, the following tree has 3 leaves and 1 vertex of degree at least 3, and $3 \geq 1$.



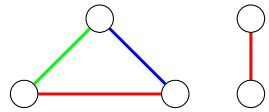
P2) (20 points) Given a connected undirected graph $G = (V, E)$ with n vertices and $m = n + k$ edges. Design an $O(m + n)$ time algorithm that outputs k edges $e_1, \ldots, e_k$ of G such that if we delete all of these edges G still remains connected. For example in the following graph if you delete both of the red edges the graph remains connected.



P3) (20 points) Given a weighted graph $G = (V, E)$ where every edge $e \in E$ has a weight $w_e \in \{1, 2, 3\}$ and a vertex $s \in V$. Design an $O(m + n)$ time algorithm that outputs the length of the shortest path from s to all vertices of V. Recall that in a weighted graph the length of a path P with edges $e_1, \ldots, e_k$ is $w_{e_1} + \cdots + w_{e_k}$. For example, in the following graph the length of the shortest path from s to a, b, c are $2, 1, 3$ respectively.



P4) (20 points) Given an undirected graph $G = (V, E)$ with n vertices such that the degree of every vertex of G is at most k. Show that we can color the edges of G with at most $2k - 1$ colors such that any pair of edges e, f which are incident to the same vertex have distinct colors. For example, in the following graph, we have $k = 2$, and we can color edges of G with $2k - 1 = 3$ colors as follows:



P5) **Extra Credit:** Prove that we can color the edges of every graph G with two colors (red and blue) such that, for every vertex v, the number of red edges touching v and the number of blue edges touch v differ by at most 2.