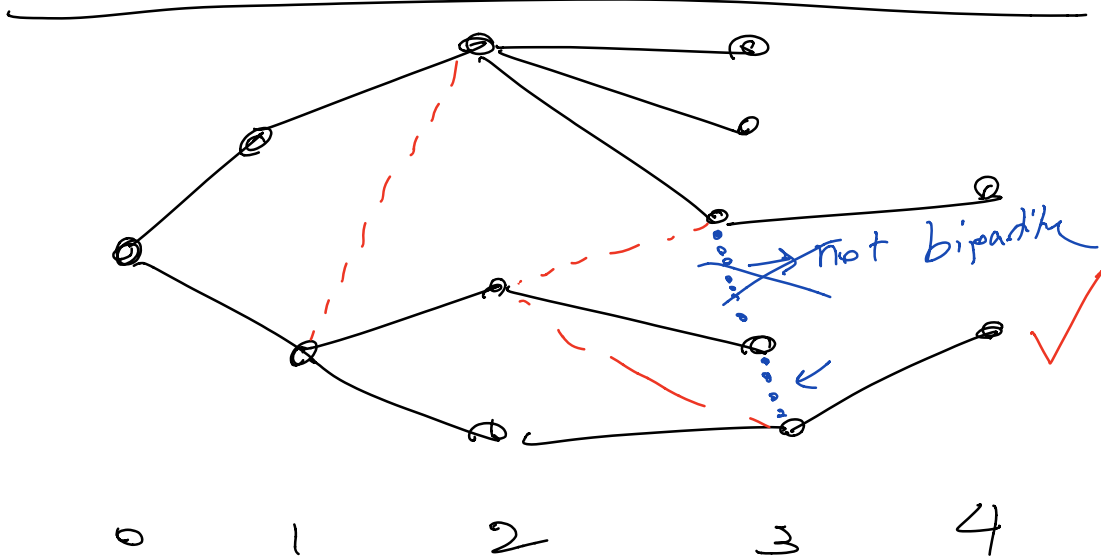


Graph G , $\deg(v) \leq k$ (n vertices)
 Color with $k+1$ colors. Assume $k \geq 1$
 Fixed param $k \geq 1$

Base Case ($n=1$):
 $k = \max \text{deg}$ of my current
 $k \geq \max \text{deg}$ of current graph \checkmark



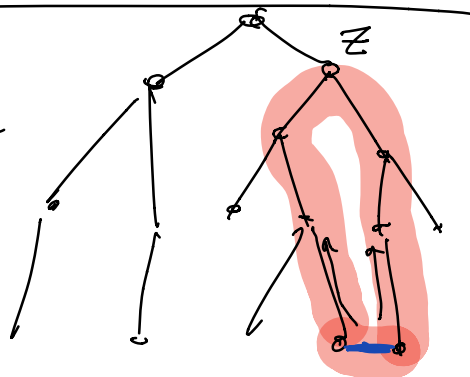
\Rightarrow Bipartite?

Claim: If \exists an edge (x,y)
 with $L(x) = L(y)$ in BFS tree
 then G has an odd cycle

Pf: Let z be the lowest common
 ancestor of x,y .

Say $L(x) = L(y) = j$ $L(z) = i$.

Then $x \dots z \dots y \dots x$ is a cycle of length $(j-i) + (j-i) + 1$
 $= 2(j-i) + 1$ odd.



No repetition BC z lowest common ancestor

OBS: During DFS(x) any newly discovered node is a descendant of x.

LEM: Any non tree edge (x,y) is ancestor-descendant

PF: Say x is discovered before y. So we run DFS(x). Good: y will be a descendant of x.

By OBS: y will be discovered during DFS(x).

If not, when we process (x,y) , y is still un-discovered but then (x,y) will be a tree edge contradiction!

