m_1: $w_1 > w_2 > w_3$

m_2: $w_2 > w_1 > w_3$

m_3: $w_3 > w_1 > w_2$

w_1: $m_3 > m_2 > m_1$

w_2: $m_1 > m_3 > m_2$

w_3: $m_2 > m_1 > m_3$

valid$(m_1) = \{w_1, w_2\}$

valid$(m_2) = \{w_1, w_2, w_3\}$

valid$(m_3) = \{w_3, w_1\}$

If w_2 is a valid pat of m_2?

yes (m_2, w_2) (m_1, w_2) (m_3, w_3) stable

Claim: Output of GS (S^*) is man optimal.

Lemma: In GS ALG no man is ever rejected from a valid partner

Lem $\Rightarrow$ Claim: BC Man go down their list so every man must be matched to his most favorite valid partner (note no-one can get matched to an invalid partner).

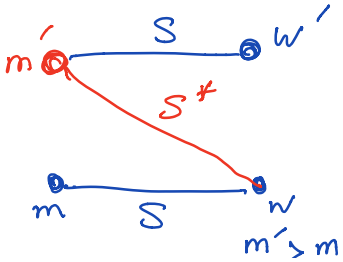
Pf of Lem. By contradiction.

Supp m is first man who is rejected by valid partner.

Supp w rejects m BC of m'

Since w is valid partner of m, $\exists$ stable matching S s.t. $(w, m) \in S$

Say w' is partner of m' is S



Case 1: $(w \succ_{m'} w')$. (m', w) unstable for S.

contradiction

Case 2: $(w' \succ_{m'} w)$ m' is already rejected by valid partner w' in GS. ⇒ contradiction! BC m was first man rejected by a valid partner