Claim: \( G \) has indup set size \( \geq k \) if \( G' \) has clique size \( \geq k \).

Pf. \( G \) has indup set size \( \geq k \) if \( G' \) has clique size \( \geq k \).

If \( S \) is a clique in \( G \), then \( G' \) has clique size \( \geq k \).

If \( S' \) is a clique in \( G' \), then \( G \) has indup set size \( \geq k \).

Vertex Cover \( \leq_b \) Indep set

Vertex Cover Given \( G, k \) does it have vertex cover \( S \), with
Claim: $S$ is a vertex cover of $G$ iff $V-S$ is an independent set of $G$.

Implies $G$ has vertex cover $\iff$ G has independent set $|S| \leq \frac{1}{2}n$

Proof of claim: $S$ vertex cover $\implies V-S$ independent set.

By contradiction: if $e \in V-S$ then $e$ is not covered by $S$ so $S$ is not a vertex cover.

$V-S$ independent set $\implies S$ vertex cover.

By contradiction: If $S$ not a vertex cover $\implies$ there is an edge not covered by $S$. So both endpoints are in $V-S$. So $V-S$ not independent set.

HW8-P1: min vertex cover is polytime in Bipartite graphs $\implies$ max independent set is also polytime solvable in Bipartite graphs.

Set Cover: Given ground elements $U$, $S_1, S_2, \ldots, S_m \subseteq U$ and integer $k$, is there $\leq k$ sets to cover all $U$. Vertex Cover $\leq_p$ Set Cover.
Claim: \( G \) has vertex cover \( 1 \leq k \leq \text{deg} \) iff \( U, k, S_v, \overline{S_u} \) is yes there are \( k \) sets to cover \( U \).

Proof: \( G \) has vertex cover \( 1 \leq k \leq \text{deg} \)
\[ S \leq V \]
\[ U, k, S_v, \overline{S_u} \text{ is yes} \]

1. Just include \( S_v \) for all \( v \in E \).
2. \( 1 \leq k \Rightarrow \) at most \( k \) sets
3. \( S \) covers all edges, and \( S_v \) has edges adjacent to \( v \).

\( U, k, S_v, \overline{S_u} \text{ is yes} \Rightarrow G \) has vertex cover \( 1 \leq k \leq \text{deg} \).

Support \( \{S_v, \overline{S_u}\} \) covers all of \( U \). Then \( v_i, v_k \) form a vertex cover of \( G \).