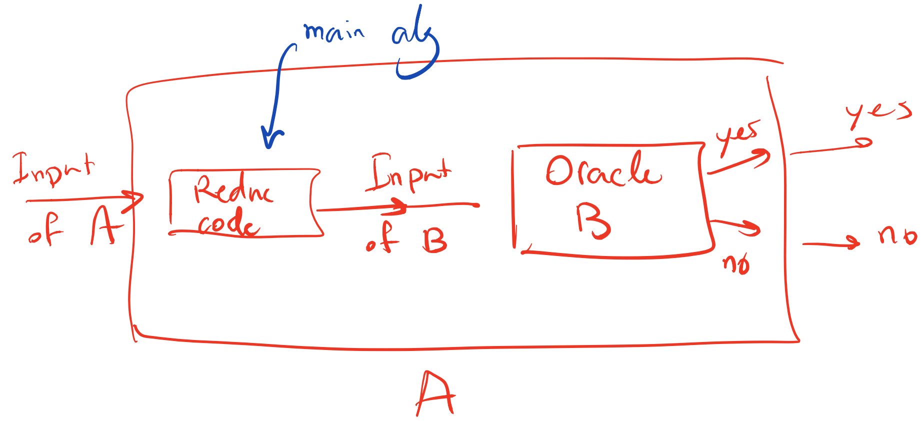
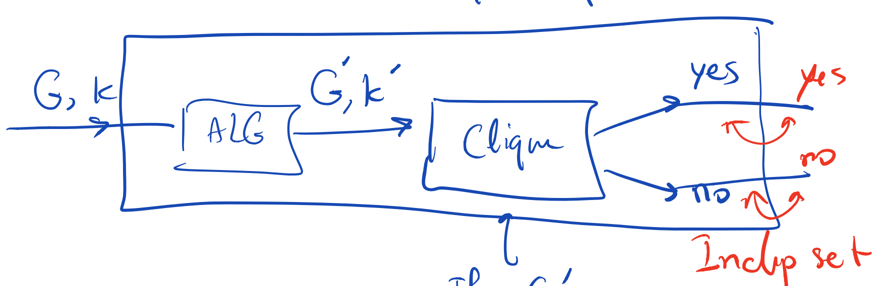


Indep set $\leq_p$ Clique



If G' has a clique of size $\geq k'$

$\begin{cases} G' = \text{comp of } G: (u,v) \text{ edge of } G' \text{ iff } (u,v) \text{ not edge of } G. \\ k' = k \end{cases}$

in poly time.

Claim: G has indep set size $\geq k$ iff G' has clique size $\geq k$.

Pf. G has indep set $|S| \geq k \Rightarrow$ G' has clique $|S'| \geq k$.

If S indep set in G then S is a clique in G' $\Rightarrow$ let $S' \leftarrow S$

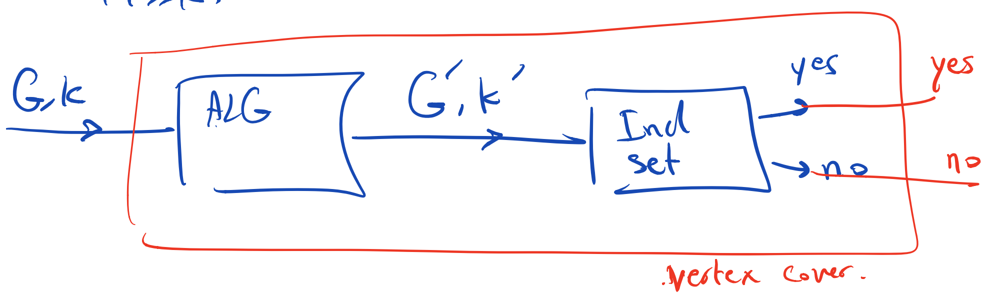
G' has a clique $|S'| \geq k \Rightarrow$ G has indep set $|S| \geq k$.

$S = S'$

Vertex Cover $\leq_p$ Indep set

Vertex Cover Given G, k does it have vertex cover S, with

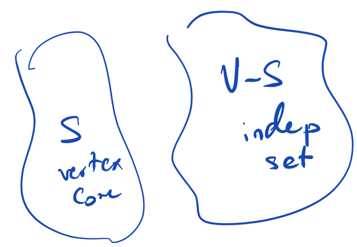
$|S| \geq k$.



$G' = G, \quad k' = n - k$

Claim: S is a vertex cover of G iff $V-S$ is an indep set of G.

Implies G has vertex cover $|S| \leq k$ $\Longleftrightarrow$ G has indep set $|S| \geq n-k$



Pf of claim: S vertex cover $\Rightarrow$ $V-S$ indep set.

$\Rightarrow$ By contradict: if $\exists e \in V-S$ then e is not covered by S so S is not a vertex cover.

$V-S$ indep set $\Rightarrow$ S vertex cover.

$\Leftarrow$ By contradiction: If S not a vertex cover $\Rightarrow \exists e$ not covered by S. So both endpoints are in $V-S$. So $V-S$ not indep set

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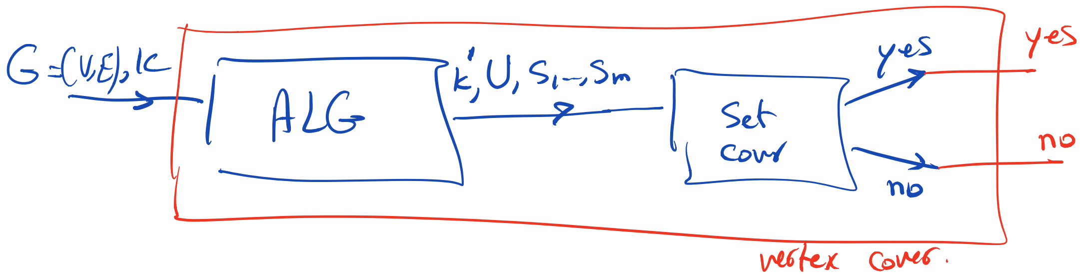
HW8-P1 min vertex cover is polytime in Bipartite graphs

$\Rightarrow$ max indep set is also polytime solvable in Bipartite graphs.

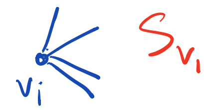
Set Cover: Given general elmts U, $S_1 \dots S_m \subseteq U$ and integ k, is there $\leq k$ sets to cover all U.

Vertex Cover $\leq_p$ Set Cover



$U = E,\quad k' = k \qquad V = \{v_1 - v_n\}$

$S_{v_i} = \{ e \in E : e \text{ incident to } v_i \}$



Claim: G has a vertex cover $|S| \leq k$ iff $U, k, S_{v_1} - S_{v_n}$ is yes; there are $\leq k$ sets to cover U.

Pf: G has vertex cover $|S| \leq k$, $S \subseteq V$ $\Rightarrow$ $U, k, S_{v_1} - S_{v_n}$ is yes

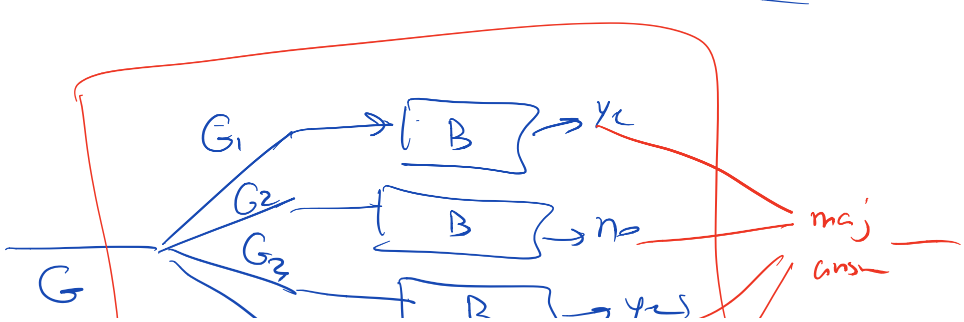
Just include S_{v_i} for all $v_i \in S$.

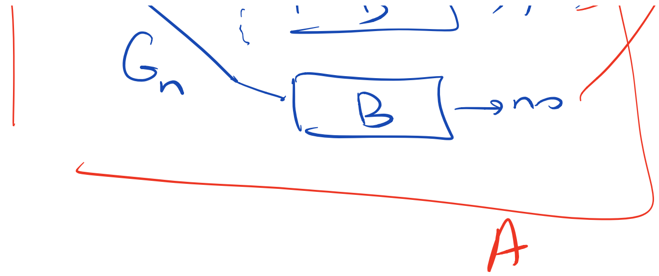
$|S| \leq k \Rightarrow$ at most k sets

S covers all edges, and S_{v_i} has edges adj to v_i implies chosen sets cover $U = E$

$U, k, S_{v_1} - S_{v_n}$ is yes $\Rightarrow$ G has vertex cover $|S| \leq k$

Supp $\{S_{v_1} - S_{v_k}\}$ covers all of U. Then $v_1 - v_k$ form a vertex cover of G.





Gn
B
→ no
A