Claim: $G = (X, Y)$ If $|N(S)| > 1$ for all $S \subseteq X$ then $|X| = |Y|$. $G$ has a perfect matching.

Pf: (By contradiction) Suppose $G$ has no perfect matching. We will find some $S \subseteq X$ s.t. $|N(S)| < |S|$. 

$|X| > \max$ Matching $G = \max$ flow $= \min$ cut $H$

Let $(A, B)$ be min s-t cut of $H$

$\text{SEA, } tEB, \: X_A = XNA, \: Y_A = YNA, \: \text{cap}(A, B) < |X|$. 

Claim: No edge $X_A - Y_B$

any such edge has infit cap.

B/c cap $(A, B)$ is finite, it does not exist.

$\text{cap} (A, B) = |X_B| + |Y_A|$  

$|N(X_A)| < |Y_A| = \text{cap}(A, B) - X_B < |X| - |X_B| = |X_A|$  

In HW8-P1, you will do similar arg.