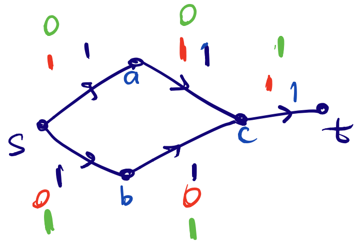
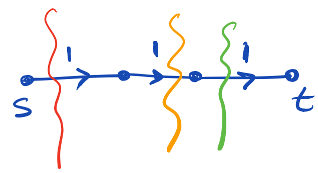
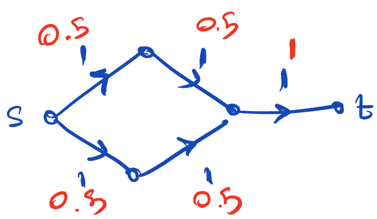
1) Given G, Is max-flow unique? no



2) Is Min (s-t) Cut unique? no



3) Is Max flow always integral? no (assuming all capacities are integral)



Claim Supp no more augmentation for $f \Rightarrow \exists$ s-t cut (A,B) s.t. $\mathrm{cap}(A,B) = v(f)$.

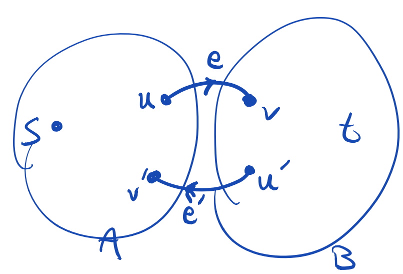
$A = \{ v : v \text{ is reachable from } s \text{ in } G_f \}$. $B = \bar{A}$

$s \in A$ (since s is reachable from s)

$t \in B$ B/C no more augmentation for f.

No edge from $A \to B$ in G_f

G

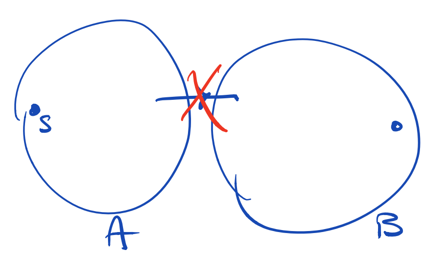


$e \notin G_f \Rightarrow c_e = f(e)$

$e' \notin G_f \Rightarrow f(e') = 0$

[If not $e'^R \in G_f$ and contradiction]

G_f



Flow value Lemma

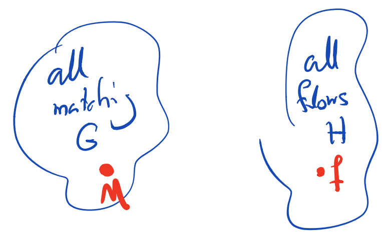
$$v(f) = \sum_{e \text{ out of } A} \overbrace{f(e)}^{c_e} - \sum_{e' \text{ into } A} \overbrace{f(e')}^{0} = \sum_{e \text{ out of } c_e} = cap(A,B).$$

max matching G = max flow H.

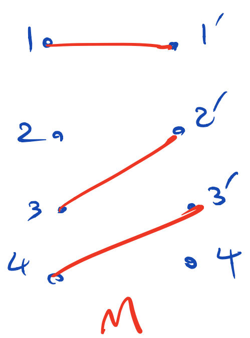
i) max matching G $\leq$ max flow H.

pf. Technique start with M to be max matching of G.

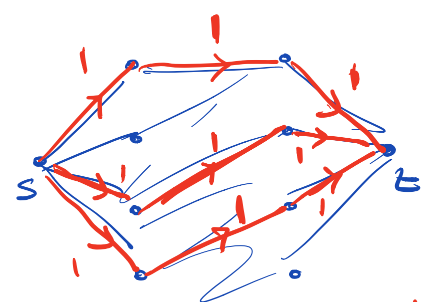
Construct a flow f s.t. $v(f) = |M|$



$\Rightarrow$ max matching G $\leq$ max flow of H.



$\Rightarrow$



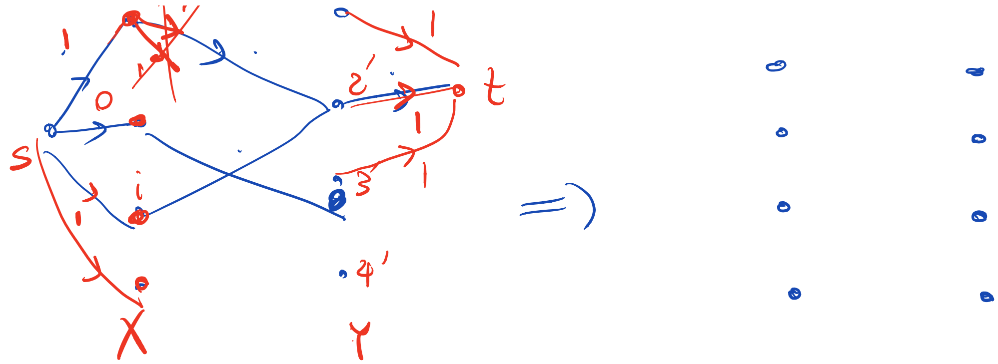
feasible flow: B/C in-flow = out flow for all excep s,t

$v(f) = |M|$.

max flow $\leq$ max matching.

Say f is an integer max-flow $\Rightarrow$ Construct a matching M st. $|M| \geq v(f)$.

(we proved it always exist)



= f integ $\Rightarrow$ $f((s,i)) = 0/1$ for all $i \in X$

$f((i',t)) = 0/1$ for all $i' \in Y$

So every $i \in X$ has either 0 or 1 incoming flow

$\Rightarrow$ either 0/1 edges with flow 1 leave i.

$\Rightarrow$ Define $M = (i, j')$ s.t $f((i,j')) = 1$

This is a matching B/C ey vertex in X or Y has at most 1 incoming or outgoin flow.

$\Rightarrow$ Also $|M| = v(f)$