

Problem: prove that for any  $n \geq 12$ , we can change  $n$  cents of postage using 4/5 cents stamps.

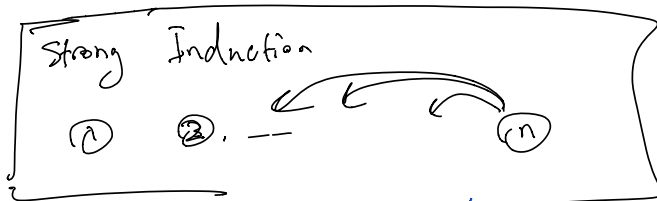
Pf.

Base Case:  $n = 12 = 4 + 4 + 4$ , 13, 14, 15 ✓

IH: We can change  ~~$n-1$~~  cents using 4/5 cent stamps for  $n-1 \geq 15$ . any amount  $12 \leq k \leq n-1$

IS: Goal make a change for  $n$ .

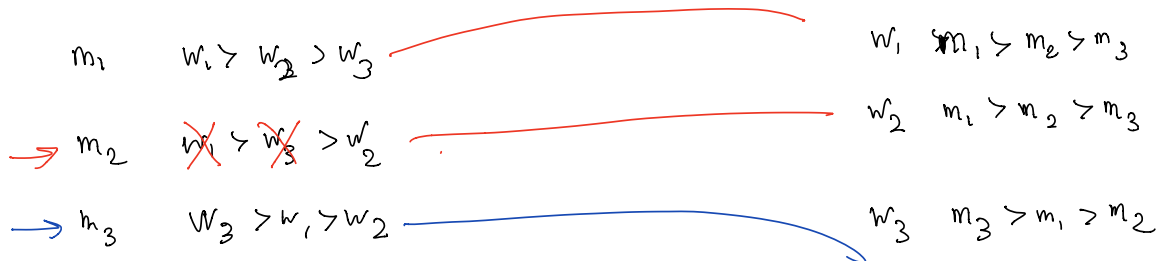
start with a 4-cent stamp then use IH to change  $n-4$



Ex.  $12 \leq n-4 \leq n-1$   
 not necessarily true  
 $n-1 \geq 15 \Rightarrow n-4 \geq 12$

Therefore by IH,  $P(n-4)$  holds and we are done.

G-S ALG example



Claim: All men and women gets matched.

Pf Pf by contradiction

Supp  $\exists$  man  $m_1$  who is not matched.

since there are  $n$  men &  $n$  women  $\Rightarrow \exists$  unmatched woman  $w_1$

BC while loop ends,  $m_1$  has proposed and get rejected by all women

$\Rightarrow m_1$  has proposed to  $w_1$

$\Rightarrow w_1$  was engaged at some point

$\Rightarrow$  But by OBS 3,  $w_1$  must be matched at end  
contradiction! D

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$\exists$  there exists  $\forall$  for all.

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Claim: There is no unstable pair.

Pf. By contradiction!

Suppose  $m_1, w_1$  is unstable

Case 1:  $m_1$  proposed to  $w_1$

$m_1$  not matched  $\Rightarrow w_1$  has rejected  $m_1$ .

$\Rightarrow$  By OBS  $w_1$  prefers her match to  $m_1$  contradiction!

Case 2:  $m_1$  never proposed to  $w_1$

by OBS 1,  $m_1$  goes down his list

$\Rightarrow$  he is matched to someone he likes more than  $w_1$

So  $m_1, w_1$  is stable contradiction!

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