Problem: prove that for any $n \geq 12$, we can change n cents of postage using 4/5 cents stamps.

Pf.

Base Case: $n = 12 = 4 + 4 + 4$, 13, 14, 15 ✓

IH: We can change ~~n-1~~ cents using 4/5 cent stamps for $n-1 \geq 15$. any amont $12 \leq k \leq n-1$

IS: Goal make a change for n.

start with a 4-cents stamp then use IH to change $n-4$

Strong Induction

1 2 . -- n

Q. $\underbrace{12 \leq n-4}_{\text{not necessarily true}} \leq n-1$

$n-1 \geq 15 \Rightarrow n-4 \geq 12$

Therefore by IH, $P(n-4)$ holds and we are done.

G-S ALG example

m_1 $w_1 > w_2 > w_3$ — w_1 $m_1 > m_2 > m_3$

→ m_2 $\cancel{w_1} > \cancel{w_3} > w_2$ — w_2 $m_1 > m_2 > m_3$

→ m_3 $w_3 > w_1 > w_2$ — w_3 $m_3 > m_1 > m_2$

Claim: All men and women gets matched.

Pf Pf by contradiction

Supp $\exists$ man m_1 who is not matched.
Since there are n men & n women $\Rightarrow$ $\exists$ unmatched woman w_1

BC while loop ends, m_1 has proposed and get rejected by all women
$\Rightarrow$ m_1 has proposed to w_1
$\Rightarrow$ w_1 was engaged at some point
$\Rightarrow$ But by OBS 3, w_1 must be matched at end
contradiction! $\square$

$\exists$ there exists $\forall$ for all.

Claim: There is no unstable pair.
Pf. By contradiction!
Suppose m_1, w_1 is unstable
Case 1: m_1 proposed to w_1
m_1 not matched $\Rightarrow$ w_1 has rejected m_1.
$\Rightarrow$ By OBS w_1 prefers her match to m_1 contradiction!

Case 2: m_1 never proposed to w_1
by OBS 1, m_1 goes down his list
$\Rightarrow$ he is matched to someone he likes more than w_1
So m_1, w_1 is stable contradiction!
